FYUG Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MATDSM–151T

(Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Define limit of a function at $x = a$.

2. Check the continuity of

$$f(x) = \begin{cases} x^2 & \text{when } x \neq 1 \\ 2 & \text{when } x = 1 \end{cases}$$

3. Use definition to find the derivative of $f(x) = \sqrt{x},\ x > 0.$

4. Write the geometrical interpretation of Rolle's theorem along with a diagram.

5. Find the values of x at which $f(x) = 2x^3 - 21x^2 + 36x - 20$ has local maxima or minima.

6. Evaluate :

$$\underset{x \to 0}{\mathrm{Lt}} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

7. Find f_x and f_y for

$$f(x) = \tan^{-1}\left(\frac{y}{x}\right)$$

8. Find the slope of tangent to the curve $y = x^3 - 3x^2 + 9$ at $x = 1$.

9. Find the polar subtangent of $r = a(1 - \cos\theta)$.

10. Show that

$$\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$$

11. Evaluate

$$\int_0^{\pi/2} \sin^{10} x\,dx$$

12. If f is an odd function, then show that

$$\int_{-a}^{a} f(x)\,dx = 0$$

13. Write the formula to compute the area bounded by two curves $y = f_1(x)$ and $y = f_2(x)$ between $x = a$ and $x = b$.

14. Write the formula for the length of the curve $y = f(x)$ between two points having x_1 and x_2 as X-coordinates.

15. Write the formula for finding the volume of the solid of revolution formed by rotating $y = f(x)$ about X-axis between $x = x_1$ and $x = x_2$.

SECTION—B

Answer any *five* questions : 10×5=50

16. *(a)* Use definition to show that

$$\underset{x \to 2}{\text{Lt}} (3x - 4) = 2$$ 3

(b) If $y = \tan^{-1} x$, then show that $(1 + x^2) y_1 = 1$. Also show that

$$(1 + x^2) y_{n+1} + 2nx y_n + n(n-1) y_{n-1} = 0$$

Hence find the value of $(y_n)_0$. 1+3+3=7

17. *(a)* Show that a function that is differentiable at a point is also continuous at that point. Give example of a function that is continuous at a point but not differentiable at that point. Justify your answer. 3+2=5

(b) State and prove Leibnitz's theorem on successive differentiation. 5

18. *(a)* State and prove Lagrange's mean-value theorem. 5

(b) Derive the expansion of $\sin x$ in powers of x, stating the conditions under which the expansion is valid. 5

19. *(a)* Show that the largest rectangle with a given perimeter is a square. 4

(b) Evaluate : 3+3=6

(i) $\underset{x \to \pi/2}{\text{Lt}} (\sin x)^{\tan x}$

(ii) $\underset{x \to 2}{\text{Lt}} \left[\dfrac{4}{x^2 - 4} - \dfrac{1}{x - 2} \right]$

20. *(a)* If $u = \log(x^2 + y^2)$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ 3

(b) State and prove Euler's theorem on homogeneous of two variables. Use it to show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

where $\tan u = \dfrac{x^3 + y^3}{x - y}$. 1+3+3=7

21. *(a)* Find the equation of the tangent to the curve

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at any point (x_1, y_1) on it. 5

(b) If $lx + my = 1$ is normal to the parabola $y^2 = 4ax$, then prove that $al^3 + 2alm^2 = m^2$. 5

22. (a) Evaluate : 5

$$\int_0^{\pi/2} \log \sin x \, dx$$

(b) Obtain a reduction formula for

$$\int_0^{\pi/2} \sin^n x \, dx$$

where n is a positive integer. 5

23. (a) Evaluate : 4

$$\int_0^{\pi/2} \frac{x \, dx}{\sin x + \cos x}$$

(b) Obtain a reduction formula for

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx$$

where m and n are positive integers. 6

24. (a) Find the area of the segment cut off from the parabola $y^2 = 4x$ by the line $y = x$. 4

(b) Find the volume and surface of the solid generated by revolving the parabola $y^2 = 4ax$ about the axis and bounded by $x = a$. 3+3=6

25. (a) Find the perimeter of the circle $x^2 + y^2 = a^2$ using integration. 4

(b) Find the volume and surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 + \cos\theta)$ about its base. 3+3=6

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