**2024/FYUG/ODD/SEM/
STADSC–102T/145**

FYUG Odd Semester Exam., 2024

STATISTICS

(1st Semester)

Course No. : STADSC–102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define continuity of a function at a point. Show that $f(x) = x^2$ is continuous at $x = a$.

(b) Evaluate :

$$\lim_{x \to \infty} x - \sqrt{x^2 + x}$$

(c) Let $f(x) = |x|$. Show that $f(x)$ is not differentiable at $x = 0$.

2. *(a)* What is homogeneous function? State and prove Euler's theorem. 1+4=5

(b) *(i)* Show that $\lim_{x\to 0} \cos\frac{1}{x}$ does not exist. 3

(ii) Evaluate : 2

$$\lim_{x\to 0} \frac{x^2 \sin\frac{1}{x}}{\sin x}$$

OR

3. *(a)* *(i)* A function $f(x)$ is defined as

$$f(x)=\begin{cases} x^2 & , \ 0<x<1 \\ x & , \ 1\le x<2 \\ \frac{1}{4}x^3 & , \ 2\le x<3 \end{cases}$$

Check the continuity of $f(x)$ at $x=1$ and $x=2$. 4

(ii) Define removable discontinuity. 1

(b) State and prove Leibnitz's theorem. 5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) Define maxima and minima of a function $f(x)$ at $x=c$.

(b) Define convexity and concavity of a function.

(c) Define Jacobian.

5. *(a)* Solve : 5

$$xyp + y^2q = 2xy - 2x^2$$

(b) If

$$u = \frac{x^2+y^2+z^2}{x}, v = \frac{x^2+y^2+z^2}{y}, w = \frac{x^2+y^2+z^2}{z}$$

find Jacobian of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. 5

OR

6. *(a)* *(i)* Define points of inflection and singular points. 3

(ii) Show that maximum value of

$$x+\frac{1}{x}$$

is less than its minimum. 2

(b) If

$$x_1 + x_2 + x_3 = u$$
$$x_2 + x_3 = uv$$
$$x_3 = uvw$$

then find

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$$ 5

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) Define gamma and beta functions.

(c) Show that $\Gamma(n+1) = n!$.

8. (a) Show that

$$\sqrt{\pi} = 2^{2m-1} \frac{\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(2m)}$$ 5

(b) Prove that

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$ 5

OR

9. (a) Prove that

$$\int_0^1 \frac{x^2}{(1-x^4)} dx \cdot \int_0^1 \frac{1}{(1+x^4)} dx = \frac{\pi}{4\sqrt{2}}$$ 5

(b) Evaluate : 5

$$\iint \sqrt{4x^2 - y^2}\, dxdy$$

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) What is exact differential equation?

(b) Define homogeneous and non-homogeneous differential equation.

(c) Solve :

$$(x^2 - 2y)dx + (y^2 - 2x)dy = 0$$

11. (a) Find the differential equation for the following : 2+3=5

(i) $y = A\sin x + B\cos x$

(ii) $y = e^{-x}(A\cos x + B\sin x)$

(b) Solve the following : 2+3=5

(i) $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$

(ii) $\frac{dy}{dx} = \frac{6x-2y-7}{2x+3y-6}$

OR

12. (a) Solve : 5

$$(x^3 + y^3)dx - xy^2 dy = 0$$

(b) Solve : 5

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$$

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) What is linear differential equation?

(b) Define auxiliary equation of a differential equation.

(c) State degree and order of partial differential equation.

14. (a) Solve : 5

$$(D^2+4)y = x^2$$

(b) Solve : 5

$$\frac{d^2y}{dx^2}+y=\cos 2x$$

OR

15. (a) Solve : 5

$$(D^2-2D+1)y = x^2e^{3x}$$

(b) Formulate a partial differential equation by eliminating *a* and *b* from

$$z=(x+a)(y+b)$$ 5

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