FYUG Odd Semester Exam., 2024

PHYSICS

(1st Semester)

Course No. : PHYDSC–101T

(Mathematical Physics—I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define dot and cross products of two vectors.

(b) What are meant by symmetric and skew-symmetric matrices?

(c) Show that the vectors

$\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$

are parallel.

2. *(a)* Define vector triple product. Show that

$$\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$ 1+5=6

(b) Show that the vectors $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{C}=2\hat{i}+\hat{j}-4\hat{k}$ form a right-angled triangle. 4

OR

3. *(a)* Define Hermitian and skew-Hermitian matrices. Show that every square matrix can be uniquely expressed as the sum of Hermitian and skew-Hermitian matrices. 2+4=6

(b) What is diagonalization of a matrix? Diagonalize the matrix

$$A=\begin{bmatrix}\frac{4}{3} & \frac{\sqrt{2}}{3}\\ \frac{\sqrt{2}}{3} & \frac{5}{3}\end{bmatrix}$$ 1+3=4

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) What is meant by differential equation? How many types of differential equation are there?

(b) Find the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{2/3}=\left(y+\frac{dy}{dx}\right)^{1/2}$$

(c) What are meant by exact and in-exact equations?

5. Solve the following equations :

(a) $(4x^3+6xy+y^2)\frac{dx}{dy}=-(3x^2+2xy+2)$ 5

(b) $\frac{dy}{dx}+xy=2x$ 5

OR

6. *(a)* What is Wronskian? Show that e^x and e^{2x} are the linearly independent solutions of $y''-3y'+2y=0$. Find the general solution with conditions $y(0)=0$ and $y'(0)=1$. 1+5=6

(b) Solve $y''-5y'+6y=0$. 4

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Define solenoidal and irrotational fields.

(b) State and explain Gauss divergence theorem.

(c) Find gradϕ at the point (–1, –2, 1) where $\phi = x^2y + xz$.

8. (a) Define directional derivative. Find its expression. 1+4=5

(b) Define gradient of a scalar function. Explain the geometrical interpretation of gradient. 1+4=5

OR

9. (a) State and prove Stokes' theorem. 6

(b) Show that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r}$ is the position vector. 4

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) Find the expression for element of arc length in curvilinear coordinates.

(b) Write down the expression for Laplacian in orthogonal curvilinear coordinates.

(c) Write the transformation equations between rectangular coordinates and spherical polar coordinates.

11. Explain the cylindrical coordinate system. Find the expression for length and volume elements in cylindrical coordinate system. Also find the expression for gradient and divergence in this system. 3+3+4=10

OR

12. (a) Explain curvilinear coordinates. Define orthogonal curvilinear coordinates. Find the expression for unit vectors in curvilinear coordinates. 3+1+3=7

(b) Show that in spherical polar coordinates the unit vectors are mutually perpendicular to each other. 3

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) Show that $\beta(m, n) = \beta(n, m)$.

(b) Define β and Γ functions.

(c) Write down Newton's forward and backward interpolation formulae.

14. (a) Discuss Newton-Raphson method of solving an algebraic equation. Using this method, find the value of $\sqrt{8}$. 4+3=7

(b) Using Simpson's $\frac{1}{3}$ rd rule, evaluate the approximate value of $\int_0^1 \frac{dx}{1+x}$ correct up to 3 decimal point taking $h = 0 \cdot 25$. 3

OR

15. (a) Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{\lfloor \frac{p+1}{2} \lfloor \frac{q+1}{2}}{2 \lfloor \frac{p+q+2}{2}}$$

and find the value of $\lfloor \frac{1}{2}$ using this formula. 4+2=6

(b) Show that $\lfloor m+1 = \underline{\lfloor m}$. 4

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