FYUG Odd Semester Exam., 2024

MATHEMATICS

(1st Semester)

Course No. : MATDSM–101T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

   (a) Using ε-δ definition, show that

   $$\lim_{x\to 0} x\sin\frac{1}{x} = 0$$

   (b) Show that $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist.

(c) For what value of k, $f(x)$ is continuous at $x = \frac{\pi}{2}$, if

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$$

2. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) If a function is differentiable at a point, then prove that it is continuous at that point. Give an example to show that if a function is continuous at a point then it is not differentiable at that point. 3+2=5

(b) If $\log y = \tan^{-1} x$ then show that

(i) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

(ii) $(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$

2+3=5

(c) Examine the continuity and differentiability of the function $f(x) = |x| + |x - 1|$ at $x = 0$. 2+2=4

(d) (i) If $y = x^{2n}$, where n is a +ve integer, then show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5, \cdots, (2n-1)\} x^n$$ 3

(ii) If $y = x^{n-1} \log x$, then show that

$$y_n = \frac{(n-1)!}{x}$$ 3

UNIT—II

3. Answer any *two* from the following : 2×2=4

(a) Examine if Rolle's theorem applicable for the function $f(x) = \tan x$ in $[-\pi, \pi]$.

(b) State necessary condition for $f(x)$ have maxima and minima.

(c) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

4. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) State and prove Cauchy's mean value theorem. 5

(b) Evaluate $\lim_{x \to 0} (\cos x)^{\cot^2 x}$. 5

(c) Prove that the rectangle with maximum area inscribed a given circle is a square. 5

(d) Show that

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \cdots + \frac{x^n}{\lfloor n} + \frac{x^{n+1}}{\lfloor n+1} e^{\xi}$$

when $0 < \xi < x$. Hence deduce that

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \cdots$$ 3+2=5

UNIT—III

5. Answer any *two* from the following : 2×2=4

(a) State Euler's theorem on homogeneous function of degree n in two variables x and y.

(b) If $f(x, y) = \tan^{-1}\frac{y}{x}$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(c) If $f(u) = V(x, y, z)$, where V is a homogeneous function of degree n then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = n\frac{f(u)}{f'(u)}$$

6. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) If

$$f(x, y) = \begin{cases} x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}, & x \neq 0, y \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Examine if $f_{xy}(0, 0) = f_{yx}(0, 0)$. 5

(b) In the curve $x^m y^n = a^{m+n}$, show that the portion of the tangent intercepted between the axes is divided at its point of contact in two segments which are in constant ratio. 5

(c) (i) If $u = \tan^{-1}\frac{x^2 + y^2}{x + y}$, then show that

$$xu_x + yu_y = \frac{1}{2}\sin 2u$$ 3

(ii) Show that for the curve $by^2 = (x+a)^3$, the square of the subtangent varies as the subnormal. 3

(d) If $lx + my = 1$ is normal to the parabola $y^2 = 4ax$, then prove that $al^3 + 2alm^2 = m^2$. 4

UNIT—IV

7. Answer any *two* from the following : 2×2=4

(a) Prove that

$$\int_0^{2a} f(x)\,dx + \int_0^a f(x)\,dx + \int_0^a f(2a - x)\,dx$$

(b) Evaluate $\int_{-a}^{a} x\phi(x^2)\,dx$.

(c) If $I_n = \int e^{-x} \cdot x^n dx$, then show that

$$I_n = -e^{-x}x^n + n I_{n-1}$$

8. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) Prove that

$$\int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \text{ if } n \text{ is even} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1 \text{ if } n \text{ is odd} \end{cases}$$

5

(b) Evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$$ 5

(c) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$, $n > 1$, then show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. 5

(d) Evaluate

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx$$ 5

UNIT—V

9. Answer any *two* from the following : 2×2=4

(a) Write the geometrical meaning of $\int_a^b f(x)dx$.

(b) Write the perimeter and area of a circle whose centre is origin and radius is r.

(c) Write the formula for finding the volume and surface area of a solid of revolution formed by rotating $y = f(x)$ about x axis between $x = a$ and $x = b$.

10. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) Find the area above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. 5

(b) Find the length of the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. 5

(c) Find the total length of the cardioid $r = a(1+\cos\theta)$. 5

(d) Find the volume of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line. 5

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