FYUG Odd Semester Exam., 2024

MATHEMATICS

(1st Semester)

Course No. : MATDSC–102T

(Differential Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

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UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) Find the value of $\lim_{x\to 0} \frac{1}{x}\log(1+x)$.

(b) Is the function $f(x)=|x-1|$ continuous at $x=1$? Justify your answer.

(c) Find from the first principle, the derivative of $\sqrt{x}$, $x>0$.

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2. Answer *either* (a) and (b) *or* (c) and (d) :

(a) State Cauchy's necessary and sufficient condition for the existence of limit. Using Cauchy's criterion for existence, examine the existence of

$$\lim_{x\to 0} \sin\frac{1}{x}$$ 1+4=5

(b) Define continuity of a function $f(x)$ at a point $x = a$. Examine the continuity of the function

$$f(x) = \begin{cases} x\sin\frac{1}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$$

at the point $x = 0$. 1+4=5

(c) If a function $f(x)$ be continuous at $x = a$ and $f(a) \neq 0$, then prove that in a neighbourhood of $x = a$, $f(x)$ has the same sign as that of $f(a)$. 5

(d) Examine the continuity and differentiability of the function

$$f(x) = \begin{cases} x^2\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at the point $x = 0$. 5

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UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) If $y = \cos^3 x$, then find y_n.

(b) If the area of a circle increases at a uniform rate, then show that the rate of increase of the perimeter varies inversely as the radius.

(c) Find the value of $\lim_{x\to 0} \frac{x - \sin x}{x^3}$.

4. Answer *either* (a) and (b) *or* (c) and (d) :

(a) (i) If $y = x^{2n}$, where n is a positive integer, then show that

$$y_n = 2^n\{1\cdot 3\cdot 5\cdot \cdots (2n-1)\}x^n$$ 3

(ii) If $u = \sin ax + \cos ax$, then show that

$$u_n = a^n\{1 + (-1)^n \sin 2ax\}^{\frac{1}{2}}$$ 3

(b) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. 4

(c) State and prove Leibnitz's theorem on the nth derivative of the product of two functions. 1+4=5

(d) If $y = \cos(m\sin^{-1} x)$, then show that—

(i) $(1 - x^2)y_2 - xy_1 + m^2 y = 0$;

(ii) $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. 5

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UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) State Rolle's theorem and write its geometric interpretation.

(b) If $f'(x) = 0$ for all values of x in an interval, then prove that $f(x)$ is constant in that interval.

(c) For what range of values of x, $2x^3 - 9x^2 + 12x - 3$ decreases as x increases?

6. Answer *either* (a) and (b) *or* (c) and (d) :

(a) State and prove Lagrange's mean value theorem. 5

(b) What do you mean by the maximum and the minimum values of a function $f(x)$ at $x = c$? Show that the maximum value of xy subject to the condition $3x + 4y = 5$ is $\frac{25}{48}$. 1+4=5

(c) (i) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$, $0 < \theta < 1$, then find the value of θ, when $h = 1$ and $f(x) = (1-x)^{\frac{5}{2}}$. 3

(ii) Show that $\sin x > x - \frac{1}{6}x^3$, if $0 < x < \frac{\pi}{2}$. 3

(d) Expand $\sin x$ in powers of x in infinite series stating the condition under which the expansion is valid. 4

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UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Find the equation of the tangent at the point (1, −1) to the curve

$$x^3 + xy^2 - 3x^2 + 4x + 5y + 2 = 0$$

(b) Find the length of the cartesian subtangent of the curve $y = e^{-\frac{1}{2}x}$.

(c) Show that in any curve

$$\rho = \left\{\left(\frac{dx}{d\psi}\right)^2 + \left(\frac{dy}{d\psi}\right)^2\right\}^{\frac{1}{2}}$$

8. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Prove that all the points of the curve

$$y^2 = 4a\left\{x + a\sin\left(\frac{x}{a}\right)\right\}$$

at which the tangent is parallel to the x-axis lie on a parabola. 5

(b) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then show that

$$\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}$$ 5

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(c) Show that at any point on the curve $x^{m+n} = k^{m-n} y^{2n}$ the mth power of the subtangent varies as the nth power of the subnormal. 5

(d) Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at an extremity of the major axis is equal to half the latus rectum. 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define a homogeneous function of two variables x and y. Find the degree of the homogeneous function

$$f(x, y) = \tan^{-1}\frac{y}{x} + \sin^{-1}\frac{x}{y}$$

(b) Define asymptote of a curve.

(c) Prove that

$$\left(a-2, -\frac{2}{e^2}\right)$$

is a point of inflection of the curve $y = (x-a)e^{x-a}$.

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10. Answer *either* (a) and (b) *or* (c) and (d) :

(a) (i) If $u = 2\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \cot\frac{u}{2} = 0$$ 4

(ii) If $u = xyf\left(\frac{y}{x}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$$ 2

(b) Determine the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$ 4

(c) Show that the points of inflection on the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$. 5

(d) (i) Examine the curve $y = x^3 - 3x + 3$ for concavity and points of inflection, if any. 2

(ii) Find the asymptotes of the curve

$$y = \frac{-8}{x^2 - 4}$$ 3

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