

2024/FYUG/ODD/SEM/
MATDSC-101T/278

FYUG Odd Semester Exam., 2024

MATHEMATICS

(1st Semester)

Course No. : MATDSC-101T

(Higher Algebra and Trigonometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks
for the questions

UNIT—I

1. Answer any *two* of the following questions :

2×2=4

- (a) State De Moivre's theorem.
- (b) Write down the expansion of $\sin\alpha$,
 $\cos\alpha$, in terms of α .
- (c) Find the equation whose roots are
7th powers of the roots of the equation
 $x^2 - 2x \cos\theta + 1 = 0$.

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(Turn Over)



2. (a) If $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots$ (n being a positive integer) then prove that—

$$(i) a_0 - a_2 + a_4 - \dots = 2^{\frac{n}{2}} \cos \frac{n\pi}{4};$$

$$(ii) a_0 + a_4 + a_8 + \dots = 2^{n-2} + 2^{\frac{n-1}{2}} \cos \frac{n\pi}{4}.$$

$$3+4=7$$

- (b) If $\frac{\sin \theta}{\theta} = \frac{2165}{2166}$, then show that θ is approximately 3° .

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OR

3. (a) Prove that

$$\sin^2 \theta \cos \theta = \theta^2 - \frac{5}{6}\theta^4 + \dots + (-1)^{n-1} \frac{3^{2n-1}}{4 \cdot 2^n} \theta^{2n}$$

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- (b) If $(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = A + iB$ then show that—

$$(i) \tan^{-1} \frac{b_1}{a_1} + \tan^{-1} \frac{b_2}{a_2} + \dots + \tan^{-1} \frac{b_n}{a_n} = \tan^{-1} \frac{B}{A};$$

$$(ii) (a_1^2 + b_1^2)(a_2^2 + b_2^2) \dots (a_n^2 + b_n^2) = A^2 + B^2.$$

$$3+3=6$$

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(Continued)

UNIT—II

4. Answer any two of the following questions :

$$2 \times 2 = 4$$

- (a) State Gregory's series.
 (b) Separate $\sin(x+iy)$ into its real and imaginary part.
 (c) Find the value of $\log(5+4i)$.

5. (a) Show that

$$\tan^{-1}(\cot \theta \tanh \phi) + \frac{1}{2i} \log \frac{\sin(\theta + i\phi)}{\sin(\theta - i\phi)}$$

4

- (b) Find the sum to infinity of

$$\cosh x + \frac{\sin x}{1} \cosh 2x + \frac{\sin^2 x}{2} \cosh 3x + \dots$$

4

- (c) Write down the exponential values of $\sinh x$, $\cosh x$.

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OR

6. (a) If $x < \sqrt{2} - 1$ then prove that

$$2 \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \right) = \frac{2x}{1-x^2} - \frac{1}{3} \left(\frac{2x}{1-x^2} \right)^3 + \frac{1}{5} \left(\frac{2x}{1-x^2} \right)^5 - \dots$$

4

- (b) If $x = \log \tan \left(\frac{\pi}{4} + \frac{y}{2} \right)$, then prove that

$$y = -i \log \tan \left(\frac{ix}{2} + \frac{\pi}{4} \right)$$

4

- (c) Find the value of $\sinh(x+iy)$.

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(Turn Over)



UNIT—III

7. Answer any two of the following questions :
2×2=4

(a) Define equivalence relation on a set A.

(b) Construct the truth table for the following compound statement :

$$p \rightarrow \sim (q \vee p)$$

(c) Fill in the blanks so that the resulting statement is equivalent to the implication $p \rightarrow q$:

(i) If _____ then _____.

(ii) _____ only if _____.

(iii) _____ is necessary for _____.

(iv) _____ is sufficient for _____.

8. (a) Prove that every equivalence relation on a set S determines a partition of S. 5

(b) Construct the truth table for the following compound statement : 5

$$(p \vee q) \leftrightarrow [(\sim p) \wedge p] \rightarrow (q \wedge p)$$

OR

9. (a) If R_1 and R_2 be two equivalence relation on a set A, then prove that $R_1 \cap R_2$ is also an equivalence relation on A. Give one example to show that union of two equivalence relations is not an equivalence relation. 5

(b) (i) Write the inverse, converse and contrapositive of the following statement : 1+1=2

"If a triangle is equilateral, then it is isosceles."

(ii) Rewrite the following statements with universal and existential quantifiers : 1+1=2

(1) Every rational number is a real number.

(2) Not all birds can fly.

UNIT—IV

10. Answer any two of the following questions :
2×2=4

(a) Write down Cauchy-Schwarz inequality and Minkowski inequality.

(b) Form the equation whose roots are the roots of the equation $3x^4 - 4x^3 + 13x - 7 = 0$ with sign changed.

(c) If α, β, γ be the roots of the equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$, find the value of $\sum \frac{1}{\alpha^2}$.

11. (a) If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, then form the equation whose roots are

$$\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma} \quad 4$$

- (b) Solve the equation by Cardan's method $x^3 + 6x + 7 = 0$. 3

- (c) Find the equation whose roots are square the roots of the equation $x^4 - 2x^3 + 3x^2 - x + 7 = 0$. 3

OR

12. (a) If x, y, z be positive rational numbers, prove that

$$\left(\frac{x^2 + y^2 + z^2}{x + y + z}\right)^{x+y+z} \geq x^x y^y z^z \geq \left(\frac{x+y+z}{3}\right)^{x+y+z} \quad 6$$

- (b) Show that the equation $x^5 - x^4 + x^3 - 2x^2 - 3 = 0$ has three positive roots and two imaginary roots. 4

UNIT—V

13. Answer any two of the following questions : 2×2=4

- (a) What do you mean by elementary operation?

- (b) If A, B are two square matrices of order n such that $AB = I$. Show that $BA = I$.

- (c) Express the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

into a sum of symmetric and skewsymmetric matrix.

14. (a) Reduce the matrix

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

into normal form and hence find the rank of it. 5

- (b) If

$$A = \begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

show that A is non-singular. Find $\text{Adj}A$ and A^{-1} . 5

OR

15. (a) What do you mean by Gaussian elimination method? Solve the equation

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

by Gaussian elimination method. $2+4=6$

- (b) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

by using elementary transformation. 4
