TDC (CBCS) Even Semester Exam., 2024

STATISTICS

(2nd Semester)

Course No. : STSDSC/GEC–201T

(Statistical Methods)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *three* questions : 1×3=3

(a) Define random variable.

(b) If $F_X(x)$ is a distribution function, then what is the value of $\underset{x \to -\infty}{\text{Lt}} F_X(x)$?

(c) Define probability mass function (p.m.f.) of a random variable.

(d) Under what condition

$$E(XY) = E(X)E(Y)?$$

2. Answer any *one* question : 2

(a) Let X be a random variable with following probability distribution :

X :	-3	6	9
$P(X=x)$:	$1/6$	$1/2$	$1/3$

Find $E(X)$ and $E(X^2)$.

(b) Show that for a random variable X,

$$E(X^2) \geq \{E(X)\}^2$$

3. Answer *either* (a) *or* (b) : 5

(a) Define mathematical expectation. If X and Y are two random variables for which expectation exists, then prove that

$$E(X+Y) = E(X) + E(Y)$$ 1+4=5

(b) A random variable X has the following probability function :

X :	0	1	2	3	4	5	6	7
$P(X=x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k.

(ii) Evaluate $P(X<6)$, $P(X=4)$ and $P(0<X\leq 5)$. 1+4=5

UNIT—II

4. Answer any *three* questions : 1×3=3

(a) Define moment generating function (m.g.f.) of a random variable.

(b) State the relation between cumulant generating function and moment generating function.

(c) State any two properties of characteristic function.

(d) Under what condition

$$M_{X+Y}(t) = M_X(t) \cdot M_Y(t)?$$

5. Answer any *one* question : 2

(a) If X_1 and X_2 are two independent random variables, then prove that

$$\phi_{X_1+X_2}(t) = \phi_{X_1}(t) \cdot \phi_{X_2}(t)$$

(b) If X is a random variable with p.m.f.

$$P(X=x) = q^x p; \quad x = 0, 1, 2, \ldots$$

then find the m.g.f. of X.

6. Answer *either* (a) *or* (b) : 5

(a) Prove that moment generating function (m.g.f.) is not independent of change of origin and scale. Define characteristic function. 4+1=5

(b) Discuss the effect of change of origin and scale on cumulants. Find the characteristic function of the random variable having p.d.f.

$$f(x) = \theta e^{-\theta x}; \quad x > 0, \ \theta > 0$$

3½+1½=5

UNIT—III

7. Answer any *three* questions : 1×3=3

(a) Under what condition

$$f_{XY}(x, y) = f_X(x) \cdot f_Y(y)?$$

(b) Define joint density function.

(c) Write the expression for $P(X = x | Y = y)$.

(d) If $F(x, y)$ is the distribution function, then what is the value of $F(-\infty, y)$?

8. Answer any *one* question : 2

(a) Define conditional probability density function and conditional distribution function.

(b) If $f_{XY}(x, y) = e^{-(x+y)}$; $x \geq 0$, $y \geq 0$, then find the marginal densities of X and Y.

9. Answer *either* (a) *or* (b) : 5

(a) The joint p.d.f. of X and Y is given by

$$f(x, y) = 4xye^{-(x^2+y^2)}; \quad x \geq 0, \ y \geq 0$$

(i) Find $f(x)$ and $f(y)$, i.e., marginal p.d.f. of X and marginal p.d.f. of Y. 2

(ii) Are X and Y independent? 1

(iii) Find the conditional distribution of X, given $Y = y$ and Y given $X = x$. 2

(b) The joint probability distribution of two random variables X and Y is given by

$$P(X = 0, Y = 1) = \frac{1}{3}$$
$$P(X = 1, Y = -1) = \frac{1}{3}$$
$$P(X = 1, Y = 1) = \frac{1}{3}$$

Find the marginal distributions of X and Y. 2½+2½=5

UNIT—IV

10. Answer any *three* questions : 1×3=3

(a) What is the relation between mean and variance of Poisson distribution?

(b) If $X \sim \exp(\theta)$, then what is the value of $E(X)$?

(c) Write the p.m.f. of binomial distribution with parameters n and p.

(d) Fill in the blank :

The standard deviation of a Poisson variate is 3, the mean of the Poisson variate is ____.

11. Answer any *one* question : 2

(a) The mean of a binomial distribution is 40 and standard deviation is 6. Calculate n, p and q.

(b) State some properties of normal distribution.

12. Answer *either (a) or (b)* : 5

(a) Derive Poisson distribution as a limiting case of binomial distribution. Give some examples of Poisson distribution. 4+1=5

(b) Obtain the moment generating function of binomial distribution with parameters n and p. Hence obtain mean and variance. 2+1+2=5

UNIT—V

13. Answer any *three* questions : 1×3=3

(a) State Central Limit Theorem (CLT).

(b) Define Weak Law of Large Numbers (WLLN).

(c) State one application of Chebyshev's inequality.

(d) Define Laplace-de Moivre CLT.

14. Answer any *one* question : 2

(a) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

(b) Define convergence in probability.

15. Answer *either (a) or (b)* : 5

(a) State and prove Chebyshev's inequality.

(b) State and prove Bernoulli's law of large numbers.

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