TDC (CBCS) Even Semester Exam., 2024

STATISTICS

(2nd Semester)

Course No. : STSHCC–202T

(Algebra)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* questions from the following : 2×2=4

(a) If a, b, c are the roots of x^3+px^2+qx+r, then find the value of

$$\sum\frac{b^2+c^2}{bc}$$

(b) If the roots of the equation $x^3+3px^2+3qx+r=0$ are in AP, then prove that $2p^3-3pq+r=0$.

(c) Find a polynomial whose roots are –3, –1, 4, 5.

2. Answer any *one* question from the following : 6

(a) Show that an algebraic equation of degree n has n root.

(b) If sum of two roots of equation $x^3 + px^2 + qx + r$ is equal to the third root, then show that $p^3 - 4pq + 8 = 0$.

UNIT—II

3. Answer any *two* questions from the following : 2×2=4

(a) Define vector space.

(b) Define linear independence and dependence.

(c) Define subspace of a vector space.

4. Answer any *one* question from the following : 6

(a) Show that the intersection of any two subspaces w_1 and w_2 of a vector space $v(F)$ is also a subspace of $v(F)$. 6

(b) (i) State the general properties of a vector space. 2

(ii) Show that the three vectors $(1, 1, -1)$, $(2, -3, 5)$, $(-2, 1, 4)$ of R^3 are linearly independent. 4

UNIT—III

5. Answer any *two* questions from the following : 2×2=4

(a) Define orthogonal matrix and identity matrix.

(b) Define determinant of a square matrix.

(c) Define symmetric and skew-symmetric matrix.

6. Answer any *one* question from the following : 6

(a) Show that every square matrix can be uniquely expressed as a sum of symmetric matrix and skew-symmetric matrix.

(b) (i) Show that $(AB)^T = B^T A^T$. 3

(ii) If

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

then show that A is an orthogonal matrix. 3

UNIT—IV

7. Answer any *two* questions from the following : 2×2=4

(a) Define rank of a matrix.

(b) Define minors and co-factors.

(c) Define inverse of a square matrix.

8. Answer any *one* question from the following : 6

(a) (i) Prove that inverse of a matrix is unique. 3

(ii) Solve the following system of equations : 3

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$

(b) (i) If A and B be two n-rowed non-singular matrices, then prove that $(AB)^{-1}=B^{-1}A^{-1}$. 3

(ii) Show that

$$\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$$

3

UNIT—V

9. Answer any *two* questions from the following : 2×2=4

(a) Define characteristic root and characteristic vectors.

(b) Define quadratic form of a matrix.

(c) Define generalized inverse of a matrix.

10. Answer any *one* question from the following : 6

(a) State and prove Caley-Hamilton theorem.

(b) Prove that if characteristic roots of a matrix A are $\lambda_1, \lambda_2, \cdots, \lambda_n$, then characteristic roots of A^2 are $\lambda_1^2, \lambda_2^2, \cdots, \lambda_n^2$.

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