TDC (CBCS) Even Semester Exam., 2024

STATISTICS

(2nd Semester)

Course No. : STSHCC–201T

(Probability and Probability Distribution)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) Define continuous random variable and probability density function.

(b) Define cumulative distribution function. State two properties of cumulative distribution function.

(c) Define joint density function and marginal distribution function.

2. Answer any *one* of the following questions : 6

(a) A random variable X has the following probability distribution :

$X = x$:	0	1	2	3	4	5	6	7
$P(X = x)$:	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2 + k$

Find the values of k, $P(X < 6)$, $P(X \geq 6)$ and $P(2 < X < 3)$. 1+2+2+1=6

(b) Define conditional probability distribution.

A two dimensional random variable (X, Y) has the joint density

$$f(x, y) = 8xy \quad , \quad 0 < x < y < 1$$
$$= 0 \quad , \quad \text{otherwise}$$

(i) Find $P\left[X < \frac{1}{2} \cap Y < \frac{1}{4}\right]$;

(ii) Find marginal density of X and Y;

(iii) Are X and Y independent? 1+2+2+1=6

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Show that for a discrete random variable X, $E(X^2) \geq \{E(x)\}^2$, provided that the 1st two moments exist.

(b) If a and b be two constants and X be any random variable, then prove that

$$E(aX + b) = aE(X) + b$$

(c) Two unbiased die are thrown. Find the expected value of the product of the numbers of points on them.

4. Answer any *one* of the following questions : 6

(a) *(i)* If X and Y be two random variables and a and b be two constants, then prove that

$$E(aX - bY) = aE(X) - bE(Y)$$ 3

(ii) Let $X_1, X_2, \cdots, X_n$ be n random variables, then prove that

$$V\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \operatorname{cov}(X_i, X_j)$$ 3

(b) *(i)* A continuous random variable has the following p.d.f. :

$$f(x) = \frac{x}{2} \quad , \quad 0 \leq x \leq 1$$
$$= \frac{1}{2} \quad , \quad 1 < x \leq 2$$
$$= (3 - x)/2 \quad , \quad 2 < x \leq 3$$

Find $E(X)$. 3

(ii) The joint probability distribution of the random variables X and Y is given below :

$X \backslash Y$	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find $E(X)$ and $E(Y)$. 3

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Show that moment generating function is independent of change of origin but not scale.

(b) Show that characteristic function of a random variable is uniformly continuous.

(c) State the properties of cumulant generating function.

6. Answer any *one* of the following questions : 6

(a) (i) Define moment generating function. Find the moment generating function (m.g.f.) of the random variable X, having probability mass function

$$P(X = x) = \frac{1}{2^x}, \ x = 1, 2, 3, \cdots$$ 1+2=3

(ii) Define cumulant generating function. Find the effect of change of origin and scale on cumulants. 1+2=3

(b) (i) Find the characteristic function and mean of the random variable X having probability density function

$$f(x) = \theta \cdot e^{-\theta x}, \ x > 0, \ \theta > 0$$ 3

(ii) If X and Y be two random variables, then prove that—

$$V(X) = E[V(X \mid Y)] + V[E(X \mid Y)]$$ 3

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Define binomial distribution. State some properties of binomial distribution.

(b) If X be a Poisson variate with parameter 4, then find the standard deviation.

(c) Find the recurrence relation between the moments of a negative Binomial distribution.

8. Answer any *one* of the following questions : 6

(a) *(i)* Derive Binomial distribution with parameters n and p, where n be the number of independent trials and p be the probability of success in each trial. 3

(ii) Prove that for Poisson distribution, mean and variance are same. 3

(b) *(i)* Define hypergeometric distribution with parameters (N, M, n) and deduce the recurrence relation of probabilities of the distribution. 1+2=3

(ii) Show that under certain conditions, binomial distribution is a special case of hypergeometric distribution. 3

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Write the p.d.f. of normal distribution having mean μ and variance σ^2. State some properties of normal distribution.

(b) Find mean and variance of a random variable X with p.d.f. given by

$$f(x) = \frac{1}{\beta - \alpha} \quad , \quad \alpha < x < \beta$$
$$= 0 \quad , \quad \text{otherwise}$$

(c) Define exponential distribution and find its mean.

10. Answer any *one* of the following questions : 6

(a) *(i)* Show that the linear combination of normal variates is also a normal variate. 3

(ii) Derive m.g.f. of normal distribution and hence obtain its cumulant generating function. 3

(b) *(i)* Define two-parameter gamma distribution and obtain its mean and variance. 3

(ii) Prove that the exponential variate inherits the memoryless property from geometric probability law. 3

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