2024/TDC (CBCS)/EVEN/SEM/
MTMHCC–202T/230

TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MTMHCC–202T

(Differential Equation)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* questions : 2×2=4

(a) Show that $x^3 + 3xy^2 = 1$ is an implicit solution of the differential equation $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ on the interval $0 < x < 1$.

(b) Prove that $\sin x$ and $\cos x$ are solutions of the differential equation $y'' + y = 0$ and these solutions are linearly independent.

(c) Define general and particular solutions.

2. Answer any *one* question : 6

(a) *(i)* Obtain a differential equation by eliminating a and b from the equation $xy = ae^{x} + be^{-x}$. 2

(ii) Prove that if the Wronskian of two solutions of a differential equation $y''(x) + Py'(x) + Qy(x) = 0$, where P and Q are either constants or functions of x alone, is either identically zero or never zero. 4

(b) *(i)* Obtain a differential equation from the relation

$$y = a\sin x + b\cos x + x\sin x$$ 3

(ii) Prove that e^{x}, e^{-x} and e^{2x} are linearly independent solutions of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

Hence write its general solution. 3

UNIT—II

3. Answer any *two* questions : 2×2=4

(a) Solve : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

(b) Find the integrating factor of $(x^3 + y^3)\,dx - xy^2\,dy = 0$.

(c) Reduce the equation

$$\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$$

in the form of linear differential equation.

4. Answer any *one* question : 6

(a) *(i)* Solve : $x\frac{dy}{dx} = y + x\sqrt{x^2 + y^2}$ 3

(ii) Solve : 3

$$(x^3 + 3xy^2)\,dx + (y^3 + 3x^2 y)\,dy = 0$$

(b) *(i)* Solve : $x\frac{dy}{dx} + 2y = x^2 \log x$ 3

(ii) Solve : $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$ 3

UNIT—III

5. Answer any *two* questions : 2×2=4

(a) Formulate the differential equation for exponential growth and decay model.

(b) Define temperature growth model.

(c) What do you mean by lake of pollution model?

6. Answer any *one* question : 6

(a) A bacterial culture is known to grow at a rate proportional to the amount present, after an hour 1000 stands of bacteria are observed in the culture and after four hours 3000 stands of bacteria, find—

(i) an expression for the number of stands of the bacteria present in the culture at any time t;

(ii) the number of stands of the bacteria originally in the culture.

(b) Uranium disintegrates at a rate proportional to the amount present at any instant. If M_1 and M_2 grams of uranium are present at time T_1 and T_2, show that the half-life of uranium is

$$\frac{(T_1 - T_2)\log 2}{\log\left(\frac{M_2}{M_1}\right)}$$

UNIT—IV

7. Answer any *two* questions : 2×2=4

(a) Solve :

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

(b) Show that

$$(yz + 2x)\,dx + (zx + 2y)\,dy + (xy + 2z)\,dz = 0$$

is integrable.

(c) Solve :

$$yz\,dx + 2xz\,dy - 3xy\,dz = 0$$

8. Answer any *one* question : 6

(a) *(i)* Solve : 3

$$\frac{x\,dx}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}$$

(ii) Find $f(y)$ such that the equation

$$\left(\frac{yz + z}{x}\right)dx - z\,dy + f(y)\,dz = 0$$

is integrable. Hence solve it. 3

(b) (i) Solve : 3

$$\frac{dx}{dt} - y = t, \quad \frac{dy}{dt} + x = 1$$

(ii) Solve : 3

$$yz \log z\, dx - zx \log z\, dy + xy\, dz = 0$$

UNIT—V

9. Answer any *two* questions : 2×2=4

(a) Find the particular integral of $(D^2 - 2D + 5)y = 10 \sin x$, $D = \frac{d}{dx}$.

(b) Solve : $\frac{d^2y}{dx^2} + y = x^3$

(c) Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

10. Answer any *one* question : 6

(a) (i) Use method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$ 4

(ii) Reduce the following homogeneous linear differential equation

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

into linear differential equation. 2

(b) (i) Find the solution of

$$\frac{d^2y}{dx^2} - 2m\frac{dy}{dx} + (m^2 + n^2)y = 0$$

when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$. 3

(ii) Solve : 3

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12 \log x}{x^2}$$

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