

**2024/TDC (CBCS)/EVEN/SEM/
MTMHCC-201T/229**

TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MTMHCC-201T

(Real Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *two* of the following questions :

2×2=4

(a) Define infimum of a set. Find the infimum of the following set :

$$\left\{ 1 + \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$$

(b) Show that if B is countable subset of an uncountable set A , then $A - B$ is uncountable.

(c) State the completeness property of \mathbb{R} .

(2)

2. Answer any one of the following questions : 10

- (a) (i) State and prove the Archimedean property of real numbers. 5
- (ii) If A and B are countable sets, then prove that $A \times B$ is countable. 5
- (b) (i) If x and y are any real numbers with $x < y$, then prove that there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$. 5
- (ii) Define a bounded subset of \mathbb{R} . If A and B are bounded subsets of \mathbb{R} , then prove that $A \cap B$ and $A \cup B$ are also bounded. $1+2+2=5$

UNIT—II

3. Answer any two of the following questions : $2 \times 2 = 4$

- (a) Prove that a finite set has no limit point.
- (b) Give an example to show that an arbitrary union of closed sets may not be closed.
- (c) Show that the set

$$\left\{ 1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \dots \right\}$$

is neither open nor closed in \mathbb{R} .

(3)

4. Answer any one of the following questions : 10

- (a) (i) Prove that the derived set of a set is closed. 5
- (ii) Show that every open set in \mathbb{R} is a union of open intervals.
- (b) (i) State and prove Bolzano-Weierstrass theorem for sets. $1+5=6$
- (ii) If A and B are subsets of the set of real numbers, then prove that—
- (1) $A \subseteq B \Rightarrow D(A) \subseteq D(B)$
- (2) $D(A \cup B) = D(A) \cup D(B)$
- where $D(F)$ denotes the derived set of $F \subseteq \mathbb{R}$. $2+2=4$

UNIT—III

5. Answer any two of the following questions : $2 \times 2 = 4$

- (a) Show that the sequence $\left\langle \frac{1}{3^n} \right\rangle$ converges to zero.
- (b) Define a bounded sequence in \mathbb{R} . Give an example of a sequence which is neither bounded above nor bounded below.

(4)

- (c) Give an example of two sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ in \mathbb{R} such that $\langle x_n \rangle$ and $\langle y_n \rangle$ are non-convergent but—

- (i) their sum $\langle x_n + y_n \rangle$ converges;
- (ii) their product $\langle x_n y_n \rangle$ converges.

6. Answer any one of the following questions : 10

- (a) (i) Prove that every convergent sequence of real numbers is bounded. Give an example to show the converse of the above result is not true. 3+2=5

- (ii) Show that the sequence $\langle x_n \rangle$ defined by $x_n = \sqrt{n+1} - \sqrt{n}$, $\forall n \in \mathbb{N}$ is convergent. 5

- (b) (i) State and prove monotone convergence theorem. 1+5=6

- (ii) Prove that a sequence in \mathbb{R} can have at most one limit. 4

(5)

UNIT—IV

7. Answer any two of the following questions :

2×2=4

- (a) Prove that the sequence $\{\frac{1}{n}\}$ is a Cauchy sequence.

- (b) Give an example of an unbounded sequence that has a convergent subsequence.

- (c) State monotone subsequence theorem.

8. Answer any one of the following questions : 10

- (a) (i) State and prove Bolzano-Weierstrass theorem for sequences.

1+5=6

- (ii) Define a Cauchy sequence in \mathbb{R} . Prove that the sequence $\langle n \rangle$ is not a Cauchy sequence in \mathbb{R} . 1+3=4

- (b) (i) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. 6

- (ii) Show that the sequence $\langle x_n \rangle$, where

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1}$$

cannot converge. 4

(6)

UNIT—V

9. Answer any two of the following questions :

2×2=4

(a) Show that the series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$$

is not convergent.

(b) State Leibnitz test.

(c) Define conditional convergent series.
Give an example of it.

10. Answer any one of the following questions : 10

(a) (i) If a series in \mathbb{R} is absolutely convergent, then prove that it is convergent. 4

(ii) Test for convergence of the following series : 3×2=6

$$(1) \frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \dots$$

$$(2) \left(\frac{2^2}{1^2} - \frac{2}{1} \right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2} \right)^{-2} + \dots$$

(7)

(b) (i) Prove that the positive termed geometric series $1 + r + r^2 + \dots$ converges for $r < 1$, and diverges to $+\infty$ for $r \geq 1$. 5

(ii) Prove that the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}, \quad p > 0$$

converges for $p > 1$ and diverges for $p \leq 1$. 5
