TDC (CBCS) Even Semester Exam., 2024

STATISTICS

(4th Semester)

Course No. : STSHCC–401T

(Statistical Inference)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) What is unbiasedness? Give an example.

(b) Define consistency. State the sufficient conditions for consistency.

(c) Define minimum variance unbiased estimator.

2. Answer any *one* of the following questions : 6

(a) Let $x_1, x_2, \cdots, x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ.

(b) Prove that a minimum variance unbiased estimator is unique in the sense that if T_1 and T_2 are minimum variance estimators for $\gamma(\theta)$, then $T_1 = T_2$, almost surely.

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Write two properties of the estimators obtained by the method of moments.

(b) If $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, then what is the estimator of λ using the method of moments?

(c) If $X \sim$ Poisson (μ), then find the maximum likelihood estimator of μ.

4. Answer any one of the following questions : 6

(a) Let $X_1, X_2, X_3, \cdots, X_n$ be i.i.d. Bin(n, p) random variables, where both n and p are unknown. Obtain their estimates by using method of moments.

(b) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance.

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Explain the concept of level of significance.

(b) Define type-I and type-II errors.

(c) Explain the concepts of most powerful test and uniformly most powerful test.

6. Answer any *one* of the following questions : 6

(a) Given the frequency function

$$f(x, \theta) = \frac{1}{\theta}, \; 0 \leq x \leq \theta$$
$$= 0, \text{ otherwise}$$

test $H_0 : \theta = 1$ against $H_1 : \theta = 2$. The sample size is one. Find out type-I and type-II errors if the critical region is *(i)* $0 \cdot 5 \leq x$ and *(ii)* $1 \leq x \leq 1 \cdot 5$.

(b) Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ for the parameter θ of the distribution

$$f(x, \theta) = \frac{1+\theta}{(x+\theta)^2}, \; 1 \leq x < \infty$$

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) State Neyman-Pearson lemma. Also write one purpose of the lemma.

(b) Write two properties of likelihood ratio test.

(c) Define likelihood ratio test.

8. Answer any *one* of the following questions : 6

(a) Let $X_1, X_2, \cdots, X_n$ be i.i.d., Bin(l, p) random variables and let $H_0: p = p_0$, $H_1: p = p_1 \, (p_1 > p_0)$. Use Neyman-Pearson lemma to find the most powerful test of size α for testing H_0 against H_1.

(b) Let X_1 and X_2 be $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ respectively, where the means and variance are unspecified. Develop likelihood ratio test for testing $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$.

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) What is interval estimation? Give example.

(b) What is the shortest length confidence interval?

(c) What are confidence interval and confidence limits?

10. Answer any *one* of the following questions : 6

(a) Deduce $1-\alpha$ level confidence interval for the binomial proportion.

(b) Obtain $100(1-\alpha)\%$ confidence limits (for large samples) for the parameter λ of the Poisson distribution

$$f(x, \lambda) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}; \; x = 0, 1, 2, \cdots$$

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