2024/TDC (CBCS)/EVEN/SEM/
PHSHCC–401T/090

TDC (CBCS) Even Semester Exam., 2024

PHYSICS

(4th Semester)

Course No. : PHSHCC–401T

(Mathematical Physics—III)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

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UNIT—I

1. Answer any *two* questions : 2×2=4

(a) Define modulus of a complex number. Find the modulus of $\frac{1-i}{1+i}$.

(b) What do you mean by argument of a complex number? Find the argument of $-3i$.

(c) Write down the Euler's formula.

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2. Answer *either* (a) *or* (b) : 6

(a) Show that $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$ and $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$. Also express the complex number $\left(\frac{2+i}{3-i}\right)^2$ into polar form. 3+3=6

(b) State and prove the necessary conditions for a function $f(z) = u + iv$ to be analytic at all the points in a region R. 6

UNIT—II

3. Answer any *two* questions : 2×2=4

(a) Define limit and continuity of a complex function.

(b) What are singular functions? Give examples.

(c) What are simple poles and poles of order n?

4. Answer *either* (a) *or* (b) : 6

(a) State and prove Cauchy's integral formula. 6

(b) Find the following function in Taylor series :

$$f(z) = \frac{1}{z+1}$$

about $z = 1$. Also evaluate $\oint_C \frac{\cos z}{z} dz$, where C is an ellipse given by $9x^2 + 4y^2 = 1$. 3+3=6

UNIT—III

5. Answer any *two* questions : 2×2=4

(a) What are residues? Find the residue of the complex function $f(z) = \frac{1}{1+z^2}$ at the pole $z = i$.

(b) State Cauchy's residue theorem.

(c) Find the residue of the complex function $e^{1/z}$.

6. Answer *either* (a) *or* (b) : 6

(a) Find the residue of the function $f(z) = \frac{e^{iz}}{z^2 + a^2}$ for the upper half circular contour. Also find the residue of a function $f(z) = \frac{z^2}{(z+1)^2 (z-2)}$ at its double pole. 3+3=6

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(b) By residue theorem, show that

$$\int_{-\infty}^{+\infty} \frac{\cos x}{x^2+1} = \frac{\pi}{e}$$

Also evaluate the following integral using calculus of residues : 3+3=6

$$\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$$

UNIT—IV

7. Answer any *two* questions : 2×2=4

(a) Define Laplace transform of a function.

(b) What are the conditions for Laplace transform to exist?

(c) Give the formula for Laplace transformation of derivative of a function and explain relevant terms.

8. Answer *either* (a) *or* (b) : 6

(a) Find the Laplace transform of the function $f(t) = t^n$. Also show that Laplace transform of integral of $f(t)$, i.e.

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}F(s)$$

where $L[f(t)] = F(s)$. 3+3=6

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(b) Find the Laplace transform of $f(t) = \sin \omega t$. Also show that Laplace transform of $e^{\alpha t}\sin\omega t$ is $\frac{\omega}{(s-\alpha)^2+\omega^2}$. 3+3=6

UNIT—V

9. Answer any *two* questions : 2×2=4

(a) Define inverse Laplace transform.

(b) Find the inverse Laplace transform of $\frac{6}{s^2+36}$.

(c) Show that Laplace transform of derivative of $f(t)$ corresponds to multiplication of the Laplace transform of $f(t)$ by s.

10. Answer *either* (a) *or* (b) : 6

(a) State and explain convolution theorem. Also find $L^{-1}\left\{\frac{1}{s(s^2+9)}\right\}$ by using convolution theorem. 3+3=6

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(b) Solve the following differential equation using Laplace transform :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

given $y(0)=2$; $y'(0)=0$. Also using Laplace transform, find the solution of the initial value problem

$y''-4y'+4y=64\ \sin 2t;\ y(0)=0,\ y'(0)=1$

3+3=6

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