

**2024/TDC (CBCS)/EVEN/SEM/  
MTMSEC-401T (A/B/C)/236**

**TDC (CBCS) Even Semester Exam., 2024**

**MATHEMATICS**

**( 4th Semester )**

Course No. : MTMSEC-401T

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

Candidates have to answer *either* from Option—A or  
Option—B or Option—C

OPTION—A

Course No. : MTMSEC-401T (A)

**( Graph Theory )**

UNIT—I

1. Answer any *three* of the following : 1×3=3

- (a) Define a graph.
- (b) Define a simple graph.
- (c) What is a complete graph?
- (d) What is a bipartite graph?

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2. Answer any one of the following : 2  
 (a) Define isolated and pendant vertices.  
 (b) Sketch the complete graph  $K_5$ .

3. Answer either (a) or (b) : 5  
 (a) Define a complete bipartite graph. Show that a complete bipartite graph  $K_{a,b}$  has  $ab$  edges.  
 (b) Show that the sum of the degrees of the vertices of a graph is twice the number of its edges.

## UNIT—II

4. Answer any three of the following :  $1 \times 3 = 3$   
 (a) Define cut point.  
 (b) What is a tree?  
 (c) What is a leaf in a tree?  
 (d) What is a forest?
5. Answer any one of the following : 2  
 (a) Show that in a tree, there is exactly one path between every pair of its vertices.  
 (b) What is a bridge in a connected graph? Illustrate with a suitable example.

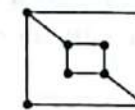
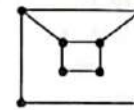
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6. Answer either (a) or (b) : 5  
 (a) "A connected graph  $G$  is bipartite if and only if  $G$  has no odd cycles." Prove it.  
 (b) Show that a tree with  $n$  vertices has  $n - 1$  edges.

## UNIT—III

7. Answer any three of the following :  $1 \times 3 = 3$   
 (a) Define Hamiltonian graph.  
 (b) Define Eulerian graph.  
 (c) Define isomorphism between two graphs.  
 (d) What is the order of the adjacency matrix of a graph with 4 vertices?
8. Answer any one of the following : 2  
 (a) Define traversable graphs with suitable example.  
 (b) Define traceable graphs with suitable example.
9. Answer either (a) or (b) : 5  
 (a) Are the following graphs isomorphic? Justify your answer :



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- (b) Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.

UNIT—IV

10. Answer any *three* of the following :  $1 \times 3 = 3$

- (a) Define planar graphs.
- (b) Give an example of a non-planar graph.
- (c) For which  $n$ , the complete graph  $K_n$  is planar?
- (d) State a necessary and a sufficient condition for a graph to be planar.

11. Answer any *one* of the following : 2

- (a) Prove or disprove :  
The complete bipartite graph  $K_{3,3}$  is planar.
- (b) If  $G$  is a planar graph with no parallel edges on  $n$  vertices and  $e$  edges, where  $e \geq 3$ , then show that  $e \leq 3n - 6$ .

12. Answer *either* (a) or (b) : 5

- (a) State and prove Euler theorem on plane graphs.
- (b) Show that a graph is planar if and only if it can be embedded on a sphere.

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UNIT—V

13. Answer any *three* of the following :  $1 \times 3 = 3$

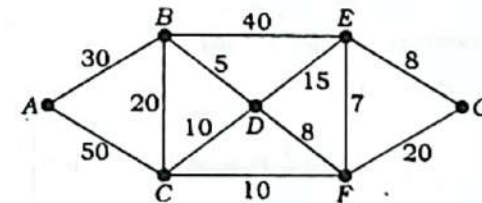
- (a) What is the travelling salesman's problem?
- (b) What is Floyd-Warshall algorithm used for?
- (c) Write one use of Dijkstra's algorithm.
- (d) Give an example of a weighted graph.

14. Answer any *one* of the following : 2

- (a) Write the steps of Floyd-Warshall algorithm.
- (b) Write the steps of Dijkstra's algorithm.

15. Answer *either* (a) or (b) : 5

- (a) Illustrate Floyd-Warshall algorithm with a suitable example.
- (b) Find a shortest path from A to G for the following graph :



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## OPTION—B

Course No. : MTMSEC-401T (B)

## ( Special Functions )

## UNIT—I

1. Answer any *three* of the following :  $1 \times 3 = 3$ 

- (a) Write down the Legendre's polynomial of first kind of order  $n$ .
- (b) Write down the differential equation of the Legendre's equation.
- (c) What are the roots of  $P_n(x) = 0$ , where  $P_n(x)$  is a Legendre's polynomial of first kind?
- (d) Fill in the blank :

$$P_n(-1) = \underline{\hspace{2cm}} .$$

2. Answer any *one* of the following :  $2$ 

- (a) Express  $2 - 3x + 4x^2$  in terms of Legendre's polynomial.

(b) Show that  $P'_n(1) = \frac{1}{2}n(n+1)$ .

3. Answer *either* (a) or (b) :  $5$ 

- (a) Prove that

$$1 + \frac{1}{2}P_1(\cos\theta) + \frac{1}{3}P_2(\cos\theta) + \dots = \log \left[ \frac{(1 + \sin \frac{\theta}{2})}{\sin \frac{\theta}{2}} \right]$$

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- (b) When  $n$  is positive integer, then prove that

$$P_n(x) = \frac{1}{\pi} \int_0^\pi [x \pm \sqrt{x^2 - 1} \cos \phi]^n d\phi$$

Hence find  $P_n(\cos\theta)$ .

## UNIT—II

4. Answer any *three* of the following :  $1 \times 3 = 3$ 

- (a) Write the Bessel's function of first kind of order  $n$ .
- (b) What is the complete solution of the Bessel's differential equation?
- (c) What is the relation between  $J_n(x)$  and  $J_{-n}(x)$ ,  $n$  being a positive integer?

- (d) Write down the Bessel's differential equation of order 0.

5. Answer any *one* of the following :  $2$ 

(a) Prove that  $J_{-\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \cdot \cos x$ .

- (b) Prove that

$$\frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x)$$

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6. Answer either (a) or (b) : 5

(a) Prove the following recurrence formulae for  $J_n(x)$  : 3+2=5

$$(i) xJ'_n = -nJ_n + xJ_{n-1}$$

$$(ii) J_{n-1} - J_{n+1} = 2J'_n$$

(b) Show that if  $n$  is a positive integer

$$J_{-n}(x) = (-1)^n J_n(x)$$

Also show that

$$J_{+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x \quad 3+2=5$$

#### UNIT—III

7. Answer any three of the following : 1×3=3

(a) Find the Laplace transform of  $t^n$ ,  $n$  is a positive integer.

(b) Write the first translation or shifting property of Laplace transform.

(c) If  $L\{F(t)\} = f(s)$ , then what is  $L\{t^n F(t)\}$ , where  $n = 1, 2, 3, \dots$  ?

(d) What is inverse Laplace transform of  $\frac{s}{s^2 - a^2}$  ?

8. Answer any one of the following : 2

(a) Evaluate  $L\{(t^2 + 1)^2\}$ ; where  $L\{F(t)\} = f(s)$ .

(b) Evaluate  $L^{-1}\left\{\frac{1}{s^2 - 2s + 2}\right\}$ , where  $L^{-1}\{f(s)\} = F(t)$

9. Answer either (a) or (b) : 5

(a) If  $F(t) = t^2$ ,  $0 < t < 4$  and  $F(t+4) = F(t)$ , then find  $L\{F(t)\}$ .

(b) Evaluate  $L^{-1}\left\{\frac{1}{s(s^2 + 9)}\right\}$ , where  $L^{-1}\{f(s)\} = F(t)$ .

#### UNIT—IV

10. Answer any three of the following : 1×3=3

(a) If  $L\{F(t)\} = f(s)$ , then what is the value of  $L\{F''(t)\}$  ?

(b) Write True or False :

An ODE is converted to an algebraic equation when we use Laplace transform.

(c) If  $L\{y(t)\} = y(s)$ , then obtain Laplace transform of  $y''(t) + y'(t) + y(t)$ .

(d) Write one application of Laplace transform in real-life problem.

11. Answer any one of the following : 2

(a) If  $u = u(x, t)$  is a function of two variables  $x$  and  $t$ , then what is (i)  $L\left\{\frac{\partial u}{\partial t}\right\}$ ,

(ii)  $L\left\{\frac{\partial^2 u}{\partial t^2}\right\}$ ;  $L[u(x, t)] = \bar{u}(x, s)$ ?

(b) Solve the following ODE by Laplace method :

$$\frac{dy}{dt} + y = 1; y(0) = 1; L\{y(t)\} = \bar{y}$$

12. Answer either (a) or (b) : 5

(a) Solve the differential equation

$$(D^2 + 9)y = \cos(2t)$$

$$\text{if } y(0) = 1, y'\left(\frac{\pi}{2}\right) = -1.$$

(b) Use Laplace transform method to solve

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$$

$$\text{if } y = 2, \frac{dy}{dt} = -1 \text{ at } t = 0.$$

#### UNIT—V

13. Answer any three of the following : 1×3=3

(a) Write the formula for infinite Fourier sine transformation of  $f(x)$ .

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(b) Write Fourier cosine integral formula.

(c) Write the relation between Laplace and Fourier transforms.

(d) Write Fourier exponential integral formula.

14. Answer any one of the following : 2

(a) If

$$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x \geq a \end{cases}$$

then find Fourier cosine transformation of  $f(x)$ .

(b) If  $f(x) = e^{-5x} + 3e^{-3x}$ , then find Fourier sine transformation of  $f(x)$ .

15. Answer either (a) or (b) : 5

(a) Using Fourier transform, show that

$$\int_0^{\infty} \frac{\cos sx}{s^2 + 1} ds = \frac{\pi}{2} e^{-x} \quad (x \geq 0)$$

(b) Using Fourier integral, show that

$$e^{-ax} = \frac{2a}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda \quad (a > 0, x \geq 0)$$

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## OPTION—C

Course No. : MTMSEC-401T (C)

## ( Vector Calculus / Vector Analysis )

## UNIT—I

1. Answer any *three* of the following :  $1 \times 3 = 3$ 

(a) What is the condition for coplanarity of three vectors?

(b) Find

$$(\hat{i} + \hat{j} - 6\hat{k}) \cdot \{(\hat{i} - 3\hat{j} + 4\hat{k}) \times (2\hat{i} - 5\hat{j} + 3\hat{k})\}$$

(c) Write the vector equation of the line passing through  $\vec{a}$  and parallel to  $\vec{b}$ .(d) What is the value of  $\vec{a} \times (\vec{b} \times \vec{c})$ ?2. Answer any *one* of the following : 2

(a) Show that

$$(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = [\vec{a}, \vec{b}, \vec{c}] \vec{c}$$

(b) Show that

$$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$$

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3. Answer *either* (a) or (b) :

5

(a) Find the vector equation of a plane in parametric form, passing through a given vector and parallel to two vectors. Also find the equation of the plane in non-parametric form.  $3+2=5$ (b) Prove the following :  $2+3=5$ 

(i)  $\vec{a} \times (\vec{b} \times \vec{a}) = (\vec{a} \times \vec{b}) \times \vec{a}$

(ii)  $(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) + (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) + (\vec{c} - \vec{d}) \cdot (\vec{a} - \vec{b}) = 0$

## UNIT—II

4. Answer any *three* of the following :  $1 \times 3 = 3$ 

(a) Define continuity of a vector function at a point.

(b) Write down the formula for  $\frac{d}{dt}(\vec{u} \cdot \vec{v})$ .

(c) Prove that the derivative of a constant vector is a zero vector.

(d) If  $\vec{r} = \sin t \hat{i} + \cos t \hat{j} + 2t \hat{k}$ , then find  $\left| \frac{d\vec{r}}{dt} \right|$ .

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5. Answer any one of the following : 2

(a) If  $\vec{r} = \vec{a} \cos nt + \vec{b} \sin nt$ , then prove that

$$\frac{d^2 \vec{r}}{dt^2} + n^2 \vec{r} = 0$$

(b) If  $\vec{A}$  and  $\vec{B}$  are differential vector functions, then prove that

$$\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

6. Answer either (a) or (b) : 5

(a) Prove that a vector function  $\vec{f}(t)$  has constant magnitude iff  $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$ .

(b) If  $\vec{r} = (3t, 3t^2, 2t^3)$ , then show that

$$[\dot{\vec{r}} \ddot{\vec{r}} \dddot{\vec{r}}] = 216$$

UNIT—III

7. Answer any three of the following : 1×3=3

- Define a scalar point function.
- Define a vector point function.
- What is the value of  $\text{grad } f$ ?
- Define an irrotational vector.

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8. Answer any one of the following : 2

(a) If  $u = x^2 - y^2 + 4z$ , then find  $\nabla^2 u$ .

(b) Show that  $\vec{\nabla} \cdot \vec{r} = 3$ .

9. Answer either (a) or (b) : 5

(a) Prove that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

(b) Find divergence and curl of

$$\text{grad}(x^3 + y^3 + z^3 - 3xyz)$$

UNIT—IV

10. Answer any three of the following : 1×3=3

(a) Write the value of

$$\int \left( 2\vec{r} \cdot \frac{d\vec{r}}{dt} \right) dt$$

(b) Define line integral of a vector function.

(c) If  $\vec{a} = 2u^4 \hat{i} - 5\hat{j} + (u - u^2) \hat{k}$ , then find  $\int \vec{a} du$ .

(d) Write the value of

$$\int \left( \vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt$$

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11. Answer any one of the following : 2

(a) Evaluate  $\int_0^2 \vec{f}(t) dt$ , if

$$\vec{f}(t) = t\hat{i} + (t^2 - 2t)\hat{j} + (3t^2 + 3t^3)\hat{k}$$

(b) Integrate the function  $\vec{F} = x^2\hat{i} - xy\hat{j}$  from (0, 0) to (1, 1) along  $y^2 = x$ .

12. Answer either (a) or (b) : 5

(a) If

$$\vec{A} = 2t^2\hat{i} + 3(t-1)\hat{j} + 4t^2\hat{k}$$

$$\vec{B} = (t-1)\hat{i} + t^2\hat{j} + (t-2)\hat{k}$$

then find—

(i)  $\int_0^2 (\vec{A} \cdot \vec{B}) dt$ ;

(ii)  $\int_0^2 (\vec{A} \times \vec{B}) dt$ . 2+3=5

(b) Prove the necessary and sufficient condition that the vector field defined by the vector point function  $\vec{F}$  with continuous derivatives be conservative is that  $\text{curl } \vec{F} = \vec{\nabla} \times \vec{F} = 0$ .

## UNIT—V

13. Answer any three of the following : 1×3=3

(a) What is the formula for tangential component of acceleration?

(b) State the principle of conservation of energy.

(c) Define kinetic energy.

(d) State the principle of work.

14. Answer any one of the following : 2

(a) A particle moves along the curve  $x = 4\cos t$ ,  $y = 4\sin t$ ,  $z = 6t$ . Find the velocity of the particle at time  $t = 0$ .

(b) Find the work done by the force  $\vec{F} = (0, 0, -mg)$  in moving a particle of mass  $m$  from  $O(0, 0, 0)$  to  $A(1, 1, 1)$  along the curve  $x = t$ ,  $y = t^2$ ,  $z = t^3$ ,  $t$  being a parameter.

15. Answer either (a) or (b) : 5

(a) A particle moves along the curve

$$x = t^3 + 1, y = t^2, z = 2t + 5$$

where  $t$  is the time. Find the magnitudes of the components of its velocity and acceleration at time  $t = 1$  in the direction  $\hat{i} + \hat{j} + 3\hat{k}$ .

(b) A moving particle of mass 3 units has position vector  $\vec{r}$  at time  $t$  as given by

$$\vec{r} = (t^2 - 3t)\hat{i} + \left(\frac{1}{2}t^2 - 1\right)\hat{j} - \frac{1}{2}t^2\hat{k}$$

Prove that the resultant force  $\vec{F}$  acting on the particle is given by

$$\vec{F} = 6\hat{i} + 3\hat{j} - 3\hat{k}$$

Also find the kinetic energy about the origin for the particle at any time  $t$ .

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