

**2024/TDC (CBCS)/EVEN/SEM/  
MTMGEC-601T/241**

**TDC (CBCS) Even Semester Exam., 2024**

**MATHEMATICS**

**( 6th Semester )**

Course No. : MTMGEC-601T

**( Differential Equation )**

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

**UNIT—I**

**1. Answer any four questions : 1×4=4**

(a) Write the condition of exactness of the differential equation  $Mdx + Ndy = 0$ , where  $M$  and  $N$  are functions of  $x$  and  $y$ .

(b) What is meant by an integrating factor?

(c) Determine the integrating factor of

$$(x^4 + y^4)dx - xy^3dy = 0$$

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(d) Is the differential equation  $x dy + y dx = 0$  exact?

(e) Solve  $\sin x \cos y dy + \cos x \sin y dx = 0$ .

2. Answer any one question :

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(a) Solve  $(y - px)(p - 1) = p$ .

(b) Show that the equation

$$(y^2 + 2xy) dx - x^2 dy = 0$$

is not exact.

3. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Solve :

$$xy(p^2 + 1) = (x^2 + y^2)p$$

(b) Solve :

$$x^2 \left( \frac{dy}{dx} \right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$$

(c) Solve :

$$y = 2px + p^2 y$$

(d) Solve :

$$(3x^2 y^4 + 2xy) dx + (2x^3 y^3 - x^2) dy = 0$$

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## UNIT—II

4. Answer any four of the following as directed :

1×4=4

(a) Write a linear ordinary differential equation of second-order with constant coefficients.

(b) Define auxiliary equation.

(c) Verify that  $\cos x$  is a solution of

$$\frac{d^2 y}{dx^2} + y = 0$$

(d) If  $y(x)$  be a solution of

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$$

satisfying  $y(x_0) = 0$  and  $y'(x_0) = 0$  for some  $x_0 \in (a, b)$ , then  $y(x) = \underline{\hspace{2cm}}$ .

( Fill in the blank )

(e) An integral belonging to complementary function of  $y'' + Py' + Qy = 0$ , where  $P, Q$  are functions of  $x$  or constants is  $\underline{\hspace{2cm}}$  if  $1 + P + Q = 0$ .

( Fill in the blank )

5. Answer any one question :

2

(a) If  $y_1(x) = \sin 3x$  and  $y_2(x) = \cos 3x$  are two solutions of differential equation  $y'' + 9y = 0$ , then show that  $y_1(x)$  and  $y_2(x)$  are linearly independent solutions.

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(b) Prove that  $y = e^x$  is a part of CF of the equation  $xy'' - (2x-1)y' + (x-1)y = 0$ .

6. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Show that linearly independent solutions of  $y'' - 2y' + 2y = 0$  are  $e^x \sin x$  and  $e^x \cos x$ . What is the general solution? 4

(b) If  $f_1(x)$  and  $f_2(x)$  are two solutions of

$$a_0(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x)y = 0$$

then prove that  $c_1 f_1(x) + c_2 f_2(x)$  is also a solution of this equation, where  $c_1$  and  $c_2$  are arbitrary constants. 4

(c) Given that  $x$ ,  $x^2$  and  $x^4$  are all solutions of

$$x^3 \frac{d^3 y}{dx^3} - 4x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} - 8y = 0$$

Show that they are linearly independent on the interval  $0 < x < \infty$  and write the general solution. 4

(d) Solve : 4

$$xy'' - (x+2)y' + 2y = 0$$

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## UNIT—III

7. Answer any four of the following as directed : 1×4=4

(a) Write a third-order homogeneous linear differential equation.

(b) Find the complementary function of  $(D^2 + 1)y = e^x \sin x$

(c) Define particular integral for the linear differential equation with constant coefficients  $f(D)y = X$ ,  $X$  is a function of  $x$  or constant.

(d) Find the particular integral of

$$\frac{d^2 y}{dx^2} - y = e^{2x}$$

(e) If  $v$  be any solution of a given non-homogeneous  $n$ th order linear differential equation and  $u$  be any solution of the corresponding homogeneous equation, then  $u+v$  is also a solution of the given non-homogeneous equation.

( Write True or False )

8. Answer any one question : 2

(a) Solve :

$$\frac{d^3 y}{dx^3} - 13 \frac{dy}{dx} - 12y = 0$$

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- (b) Find the complete solution of  
 $(D^2 + 1)^2 y = \cos 3x$

9. Answer either [(a) and (b)] or [(c) and (d)] :

- (a) Solve the initial value problem

$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 12y = 0, \quad y(0) = 3, \quad y'(0) = 5 \quad 4$$

- (b) Using the method of variation of parameter, solve

$$\frac{d^2 y}{dx^2} + 9y = \sec 3x \quad 4$$

- (c) Find the complete solution of

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \sin x \quad 4$$

- (d) Given that  $y = x$  is a solution of

$$(x^2 + 1) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

Find a linearly independent solution by reducing the order. 4

#### UNIT—IV

10. Answer any four of the following as directed :

1×4=4

- (a) What is meant by a Cauchy-Euler equation?

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( Continued )

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- (b) Write the condition for exactness of the differential equation

$$P dx + Q dy + R dz = 0$$

where  $P, Q, R$  are the functions of  $x, y, z$

- (c) Give an example of a homogeneous total differential equation.

- (d) The system of differential equations

$$\frac{dx}{dt} = 2x - y - 5t$$

$$\frac{dy}{dt} = 3x + 6y - 4$$

is an example of homogeneous system.

( Write True or False )

- (e) For  $F_1 dx + F_2 dy + F_3 dz = 0$ , write the auxiliary equations. ( $F_1, F_2$  and  $F_3$  are the functions of  $x, y, z$ )

11. Answer any one question :

2

- (a) Solve :

$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{x}$$

- (b) Reduce

$$x^2 \frac{d^2 y}{dx^2} + y = 3x^2$$

to linear differential equation with constant coefficients.

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( Turn Over )



12. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Solve :

4

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$$

(b) Solve :

4

$$\frac{dx}{dt} + 2x - 3y = t$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$

(c) Verify that the equation

$$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$$

is integrable and solve it.

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(d) Solve :

4

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

#### UNIT—V

13. Answer any four questions :

1×4=4

(a) Write a second-order linear partial differential equation.

(b) Define degree of a partial differential equation.

(c) Is the partial differential equation

$$\frac{\partial z}{\partial y} = 3 \left( \frac{\partial z}{\partial x} \right)^2$$

linear?

(d) What is the order of the partial differential equation

$$y \left\{ \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \right\} = z \frac{\partial z}{\partial y} ?$$

(e) Define non-linear partial differential equation.

14. Answer any one question :

2

(a) Find the order and degree of

$$\left( 1 + \frac{\partial^2 z}{\partial x^2} \right)^{3/2} = \left( 1 + \frac{\partial z}{\partial y} \right)^{1/2}$$

(b) Eliminate  $a$  and  $b$  from  $z = a(x+y) + b$  to form the partial differential equation.

15. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Form a partial differential equation by eliminating functions  $f$  and  $g$  from

$$z = f(x + iy) + g(x - iy)$$

4

- (b) Find the differential equation of all spheres of radius 4 with centres on the  $xy$ -plane.

4

- (c) For each of the following partial differential equations, determine whether the equation is hyperbolic, parabolic or elliptic :  $2+2=4$

(i)  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

(ii)  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$

- (d) Find a partial differential equation by eliminating  $a, b, c$  from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

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