TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(6th Semester)

Course No. : MTMSEC-601T

(Analytical Geometry)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *three* of the following as directed : 1×3=3

(a) If the origin is transferred to the point (h, k) without changing direction of the axes, then write the transformation formula for the translation of axes.

(b) What does the equation $x^2 - y^2 = 0$ become when the origin is transferred to the point $(-1, 2)$?

(c) A homogeneous second degree equation always represents a pair of straight lines passing through origin.

(Write True or False)

(d) Write down the equation of bisectors of angles between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$.

2. Answer any *one* question : 2

(a) Transform to axes inclined at 30° to the original axes the equation

$$x^2 + 2\sqrt{3}\, xy - y^2 - 2 = 0$$

(b) Show that $3x^2 + 5xy + 2y^2 = 0$ represents a pair of straight lines. Find the straight lines.

3. Answer any *one* question : 5

(a) Find the angle by which the axes should be rotated so that the equation $ax^2 + 2hxy + by^2 = 0$ becomes another equation in which the xy term is absent. Also find the angle through which the axes are to be rotated so that the equation $17x^2 + 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1$, $A > 0$. Find also A and B. 2+3=5

(b) Find the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$. Also write the condition of coincidence and perpendicularity of the lines. 3+2=5

UNIT—II

4. Answer any *three* of the following questions : 1×3=3

(a) Obtain the equation of the circle whose center is at (–1, –2) and radius is $\sqrt{2}$.

(b) Define orthogonal circles.

(c) Write down the equation of circles which intersect two circles $x^2 + y^2 + 2x + 1 = 0$ and $x^2 + y^2 + 2y + 3 = 0$.

(d) Find the radical axis of the two circles

$$x^2 + y^2 + 4x - 2y + 9 = 0$$

and

$$x^2 + y^2 + 2x + 3y - 5 = 0.$$

5. Answer any *one* question : 2

(a) Find the value of λ for which the circles

$$x^2 + y^2 + \lambda x + 3y - 5 = 0$$

and

$$x^2 + y^2 + 5x + xy + 7 = 0$$

cut each other orthogonally.

(b) Show that the straight line $lx+my+n=0$ touches the circle $x^2+y^2=a^2$ if $n^2=a^2(l^2+m^2)$.

6. Answer any *one* question : 5

(a) (i) Find the equation of the circle which passes through origin and cut orthogonally the circles

$$x^2+y^2-8y+12=0$$

and

$$x^2+y^2-4x-6y-3=0$$ 3

(ii) Find the radical center of the set of circles

$$x^2+y^2+x+2y+3=0,\ x^2+y^2+2x+4y+5=0$$

and $x^2+y^2-7x-8y-9=0$. 2

(b) If two tangents drawn from a point to a conic be perpendicular to one another, then prove that the locus of their point of intersection is a circle. 5

UNIT—III

7. Answer any *three* of the following questions : 1×3=3

(a) Define focal chord of a parabola.

(b) Find the foci of the ellipse $9x^2+25y^2=225$.

(c) Write down the condition that the line $y=mx+c$ is a tangent to the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

(d) Write the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in parametric form.

8. Answer any *one* question : 2

(a) Find the equation of the ellipse which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$.

(b) Find the points on the conic $\frac{14}{r}=3-8\cos\theta$, whose radius vector is 2.

9. Answer any *one* question : 5

(a) (i) Obtain polar equation of a conic referred to a focus as pole. 3

(ii) Write polar equation of the ellipse

$$\frac{x^2}{36}+\frac{y^2}{20}=1,$$

if the pole be at its right-hand focus and the positive direction of the x-axis be the positive direction of the polar axis. 2

(b) Prove that the straight line $lx+my+n=0$ is a normal to the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

if

$$\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$$ 5

UNIT—IV

10. Answer any *three* of the following as directed : 1×3=3

(a) Under what condition

$$ax^2+by^2+cz^2+2gx+2fy+2hz+c=0$$

represents a sphere?

(b) The section of a sphere by a plane represents a _____.

(Fill in the blank)

(c) Find the equation of the tangent plane to the sphere $x^2+y^2+z^2=49$ at the point (6, −3, −2).

(d) What is the equation of a sphere which passes through origin having intercepts a, b and c on the axes?

11. Answer any *one* question : 2

(a) Find the radius of the circle $x^2+y^2+z^2=25,\ x+2y+2z+9=0$.

(b) Find the values of c for which the plane $x+y+z=c$ touches the sphere

$$x^2+y^2+z^2-2x-2y-2z-6=0$$

12. Answer any *one* question : 5

(a) Show that the length of the shortest distance between the straight lines

$$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5} \text{ and } \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

is $4\sqrt{3}$ units and the equations of the line of shortest distance are $x=y=z$. 5

(b) (i) A plane passing through a fixed point (a, b, c) cuts the axes in A, B and C. Show that the locus of the center of the sphere $OABC$ is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$ 3

(ii) Find the equation of the circle on the sphere $x^2+y^2+z^2=49$ whose center is at the point (2, −1, 3). 2

UNIT—V

13. Answer any *three* of the following questions : $1\times3=3$

(a) What do you mean by guiding curve of a cone?

(b) Define axis and semi-vertical angle of a cone.

(c) Define axis of a cylinder.

(d) What do you mean by generator of a cylinder?

14. Answer any *one* question : 2

(a) Find the equation of the cone whose vertex is at the origin and whose axis is $\frac{x}{3}=\frac{y}{2}=\frac{z}{4}$ and semi-vertical angle 45°.

(b) Find the equation of the cylinder generated by the lines parallel to $\frac{x}{1}=\frac{y}{-2}=\frac{z}{5}$, the guiding curve being the conic $x=0$, $y^2=8z$.

15. Answer any *one* question : 5

(a) Prove that $ax+by+cz=0$ $(abc\neq0)$, cuts the cone $yz+zx+xy=0$ is perpendicular straight lines if

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

5

(b) (i) Find the equation of a cylinder whose guiding curve is represented by $f(x, y)=0$, $z=0$ and whose generators are parallel to the line

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$$

3

(ii) Find the equation of the cylinder generated by the straight lines parallel to z-axis and passing through the curve of intersection of the plane $lx+my+nz=p$ and the surface $ax^2+by^2+cz^2=1$. 2

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