TDC (CBCS) Even Semester Exam., 2024

STATISTICS

(6th Semester)

Course No. : STSHCC–602T

(Multivariate Analysis and Non-parametric Methods)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) If (X, Y) has a Bivariate Normal Distribution (BVN) with parameters μ_1, μ_2, σ_1^2, σ_2^2 and ρ, then prove that X and Y are independent if and only if $\rho = 0$.

(b) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then find the conditional distribution of X for a given Y.

(c) If (X, Y) follows Bivariate Normal Distribution with parameters $\mu_x = 5$, $\mu_y = 10$, $\sigma_x^2 = 1$, $\sigma_y^2 = 25$ and $\text{cov}(X, Y) = \rho$, then find ρ, when $P(4 < Y < 16 \mid x = 5) = 0 \cdot 954$.

2. Answer *either* (a) and (b) *or* (c) and (d) from the following questions :

(a) If X and Y are standard normal variates with coefficient of correlation ρ, then show that—

(i) $X + Y$ and $X - Y$ are independently distributed; 1½

(ii) $Q = \dfrac{X^2 - 2\rho XY + Y^2}{(1-\rho^2)}$ is distributed like a χ^2-variate. 1½

(b) Let $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then show that marginal p.d.f.'s of X and Y are also normal with $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$. 3

(c) If $(X, Y) \sim BVN(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, then write the expression for the moment generating function (m.g.f.). Also write the expression for m.g.f. when $(X, Y) \sim BVN(0, 0, 1, 1, \rho)$. 2

(d) The variables X and Y with zero means and standard deviations σ_1 and σ_2 are normally distributed with correlation coefficient ρ. Show that

$$U = \frac{X}{\sigma_1} + \frac{Y}{\sigma_2} \quad \text{and} \quad V = \frac{X}{\sigma_1} - \frac{Y}{\sigma_2}$$

are independent normal variates with variances $2(1+\rho)$ and $2(1-\rho)$ respectively. 4

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define cumulative distribution function of a p-dimensional random vector.

(b) Define variance-covariance matrix of the random vector $\underset{\sim}{X}_{p \times p}$.

(c) Define marginal density of a continuous and discrete multivariate random variable.

4. Answer *either* (a) *or* (b) from the following questions :

(a) (i) For the following matrix

$$X = \begin{bmatrix} 9 & 5 & 1 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } C = [-1, 2]'$$

evaluate the sample mean and variance of $C'X$. 3

(ii) Let $\underset{\sim}{X} = (X_1, X_2, \ldots, X_p)'$ be a vector of random variables. Define $Y_1 = X_1$, $Y_i = X_i - X_{i-1}$; $i = 1, 2, \ldots, p$. If Y_i's are mutually independent each with variance σ^2, then prove that

$$\text{tr}(\Sigma) = \sigma^2 \frac{p(p+1)}{2}$$

where Σ is the variance-covariance matrix of X. 3

(b) (i) Define multinomial distribution. Obtain the m.g.f. of multinomial distribution. 3

(ii) State the applications of multinomial distribution. 1

(iii) Let $\underset{\sim}{X}$ be a $p \times 1$ vector of random variables, $E(\underset{\sim}{X}) = \underset{\sim}{\mu}$ and Σ = variance-covariance matrix of $\underset{\sim}{X}$, then prove that $E(\underset{\sim}{X}\underset{\sim}{X}') = \Sigma + \underset{\sim}{\mu}\underset{\sim}{\mu}'$. 2

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Define multivariate normal distribution.

(b) If $\underset{\sim}{X} \sim N_3(\underset{\sim}{0}, \Sigma)$, then prove that $\underset{\sim}{X}'\Sigma^{-1}\underset{\sim}{X}$ follows χ^2-distribution with 3 d.f.

(c) Define multiple correlation coefficient with example.

6. Answer *either* (a) *or* (b) from the following questions :

(a) Let $\underset{\sim}{X}_{p \times 1} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then find the moment generating function of $\underset{\sim}{X}$. 6

(b) Let $\underset{\sim}{X}_{p \times 1} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. Partition $\underset{\sim}{X}$, $\underset{\sim}{\mu}$ and Σ into two subvectors and show that the conditional distribution is also multivariate normal. 6

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) What do you understand by expected sample size of a sequential sampling plan?

(b) Define operating characteristic (OC) function of sequential probability ratio test (SPRT).

(c) Fill in the blanks :

(i) The theory of sequential sampling plan was originally given by _____.

(ii) In sequential sampling plan, the sample size is a _____.

8. Answer *either* (a) *or* (b) from the following questions :

(a) If the SPRT of strength (α, β) with boundary points (A, B) terminates with probability 1, then prove that

$$A \leq \frac{1-\beta}{\alpha} \quad \text{and} \quad B \geq \frac{\beta}{1-\alpha}$$ 6

(b) Given the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 \ (> \theta_0)$ in sampling from a normal density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \rho^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \ -\infty < x < \infty$$

where σ is known. Obtain its ASN and OC function. 6

Unit—V

9. Answer any *two* of the following questions : 2×2=4

(a) Write the advantages of non-parametric tests.

(b) Mention the assumptions underlying the paired sign test.

(c) Write some limitations of Kolmogorov-Smirnov one-sample test.

10. Answer *either* (a) *or* (b) from the following questions :

(a) (i) What is sign test? Describe how the method is used in non-parametric tests. 3

(ii) What is run test? Show how the theory of runs may be used to test for the randomness of a sample. 3

(b) (i) Describe Wilcoxon paired sample signed rank test, stating clearly the underlying assumptions and the hypothesis. 3

(ii) Discuss Mann-Whitney U-test. 3

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