2023/FYUG/ODD/SEM/
MATDSC–102T/141

FYUG Odd Semester Exam., 2023

(Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATDSC–102T

(Differential Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting *two* from each Unit : 2×10=20

UNIT—I

1. Find the value of $\underset{x \to 0}{\text{Lt}} \left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x} \right)$.

2. Find the points of discontinuity of the function $\dfrac{x^2+2x+5}{x^2-8x+12}$.

3. Find the derivative of $\frac{\sin x}{x}$.

UNIT—II

4. Evaluate $\underset{x \to 0}{\text{Lt}} \frac{(e^x - 1)\tan^2 x}{x^2}$.

5. Find y_n if $y = \sin^3 x$.

6. Find the range of values of x for the function

$$x^3 - 3x^2 - 24x + 30$$

UNIT—III

7. Write down the Cauchy's form of remainder in Taylor's expansion.

8. Write down the geometrical interpretation of mean value theorem.

9. Is Rolle's theorem applicable for the function $f(x) = \tan x$ in $(0, \pi)$?

UNIT—IV

10. Write down the condition of orthogonality of the curves $f(x, y) = 0$, $\phi(x, y) = 0$.

11. What is the geometrical meaning of $\frac{dy}{dx}$?

12. Write down the formula for polar subtangent and polar subnormal of the curve $r = f(\theta)$.

UNIT—V

13. What do you mean by point of inflection?

14. When is a function said to be a homogeneous function?

15. What do you mean by asymptote of a curve?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 10×5=50

UNIT—I

16. *(a)* What do you mean by removable discontinuity? Show that a function which is continuous throughout a closed interval is bounded therein. 2+4=6

(b) Show that $\underset{x \to 0}{\text{Lt}} \cos\frac{1}{x}$ does not exist. 4

17. *(a)* Define continuity of a function. A function $f(x)$ is defined as

$$\begin{aligned} f(x) &= 3+2x \quad \text{for } -\tfrac{3}{2} \le x < 0 \\ &= 3-2x \quad \text{for } 0 \le x < \tfrac{3}{2} \\ &= -3-2x \quad \text{for } x \ge \tfrac{3}{2} \end{aligned}$$

Show that $f(x)$ is continuous at $x = 0$ and discontinuous at $x = \frac{3}{2}$. 2+4=6

(b) Find the derivative of x^x from first principle. 4

UNIT—II

18. *(a)* State L' Hospital theorem. Evaluate

$$\underset{x \to 0}{\text{Lt}} (\cos x)^{\cot^2 x}$$ 2+4=6

(b) If $y = \frac{1}{x^2 + a^2}$, then find y_n. 4

19. *(a)* If the area of a circle increases at a uniform rate, then show that the rate of increase of the perimeter varies inversely as the radius. 3

(b) If $y = \sin(m \sin^{-1} x)$, then show that—

(i) $(1-x^2)y_2 - xy + m^2 y = 0$;

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

2+3=5

(c) Find the nth derivative of $\frac{1}{x^2 - a^2}$. 2

UNIT—III

20. (a) Prove that

$$\underset{x \to 0}{\text{Lt}} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$$

provided $f''(x)$ is continuous. 4

(b) Expand $\log(1+x)$ in powers of x in infinite series stating each of the condition under which the expansion is valid. 4

(c) State Rolle's theorem. 2

21. (a) If $f(x) = x^2$, $\phi(x) = x$, then find a value of ξ in terms of a and b in Cauchy's mean value theorem. 3

(b) If

$$f(x) = \begin{vmatrix} \sin x & \sin\alpha & \sin\beta \\ \cos x & \cos\alpha & \cos\beta \\ \tan x & \tan\alpha & \tan\beta \end{vmatrix}, \ 0 < \alpha < \beta < \frac{\pi}{2}$$

then show that $f'(\xi) = 0$, where $\alpha < \xi < \beta$. 4

(c) In the mean value theorem

$$f(x+h)+f(x)+hf'(x+\theta h)$$

find θ, where $f(x)=\frac{1}{x}$. 3

UNIT—IV

22. (a) If $x\cos\alpha + y\sin\alpha = P$ touch the curve $\frac{x^m}{a^m}+\frac{y^m}{b^m}=1$, then show that

$$(a\cos\alpha)^{\frac{m}{m-1}}+(b\sin\alpha)^{\frac{m}{m-1}}=P^{\frac{m}{m-1}}$$ 4

(b) Find the angle of intersection of the curves $r=a\sin 2\theta$, $r=a\cos 2\theta$. 3

(c) Show that in any curve

$$\frac{\text{subnormal}}{\text{subtangent}}=\left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$ 3

23. (a) Show that the curves $r^n=a^n\cos n\theta$ and $r^n=b^n\sin\theta$ cut orthogonally. 3

(b) (i) Find $\frac{dS}{d\theta}$ of the curve $r=a(1+\cos\theta)$.

(ii) Find $\frac{dS}{dr}$ of the curve $r=a\theta$. 2+2=4

(c) Find the polar subtangent of $r=ae^{\theta\cot\alpha}$. 3

UNIT—V

24. (a) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, then show that—

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\phi\left(\frac{y}{x}\right)$

(ii) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$ 2+4=6

(b) Find the asymptotes of the curve

$$2x(y-5)^2 = 3(y-2)(x-1)^2$$

4

25. (a) State and prove Euler's theorem on homogeneous function $f(x, y)$. 1+3=4

(b) If $u = F(y-z, z-x, x-y)$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

3

(c) Find the asymptotes of the curve

$$x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$$

3

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