2023/TDC(CBCS)/EVEN/SEM/
STSHCC–602T/276

TDC (CBCS) Even Semester Exam., 2023

STATISTICS

(Honours)

(6th Semester)

Course No. : STSHCC–602T

(Multivariate Analysis and Non-parametric Tests)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. State the assumptions of deriving bivariate normal (BVN) distribution.

2. If X_1 and X_2 are standard normal variates with correlation coefficient ρ between X_1 and X_2, then show that the correlation coefficient between X_1^2 and X_2^2 is ρ^2.

3. If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then show that the linear combination of X and Y, i.e., $aX + bY$ is a normal variate.

4. If $\underset{\sim}{X}_{p \times 1}$ be a multivariate normal vector, then define the variance-covariance matrix of $\underset{\sim}{X}_{p \times 1}$.

5. Define cumulative distribution function of a discrete and continuous multivariate random variable.

6. Consider the following data matrix (X) :

$$X = \begin{bmatrix} 9 & 51 \\ 1 & 32 \end{bmatrix}$$

Let $C = (-1 \;\; 2)'$. Evaluate the sample mean and variance of $C'X$.

7. Define multivariate normal distribution.

8. If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then prove that

$$\underset{\sim}{Z} = D\underset{\sim}{X} \sim N_q(D\underset{\sim}{\mu}, D\Sigma D')$$

where D is any $q \times p$ matrix of rank $q (\leq p)$.

9. Define partial correlation coefficient and multiple correlation coefficient.

10. What is sequential sampling inspection plan?

11. What is OC function of a sequential probability ratio test (SPRT)?

12. Make comparative statement on single, double and sequential sampling.

13. Why are non-parametric tests called distribution-free tests?

14. State the assumptions associated with non-parametric tests.

15. Define the term 'Run'. State the assumptions of Run-test.

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. Define bivariate normal distribution. Obtain m.g.f. of bivariate normal distribution with parameters μ_1, μ_2, σ_1^2, σ_2^2 and ρ. 2+4=6

17. *(a)* If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then obtain the conditional distribution of Y given $X = x$ and of X given $Y = y$. 4

(b) If $(X, Y) \sim BVN(0, 0, 1, 1, \rho)$, then obtain the line of regression of Y on X. 2

18. Define multinomial distribution. If $X_1, X_2, \cdots, X_k$ are k-independent Poisson variates with parameters $\lambda_1, \lambda_2, \cdots, \lambda_k$ respectively, then prove that the conditional distribution

$$P(X_1 \cap X_2 \cap \cdots \cap X_k \mid X)$$

where $X = X_1 + X_2 + \cdots + X_k$ is fixed, is multinomial. 2+4=6

19. *(a)* Let $\underset{\sim}{X} = (X_1, X_2, \cdots, X_p)'$ be a vector of random variable. Define $Y_1 = X_1$, $Y_i = X_i - X_{i-1}$, $i = 2, 3, \cdots, p$. If Y_i's are mutually independent, each with variance σ^2, then prove that

$$\text{tr}\,(\Sigma) = \frac{p(p+1)}{2} \cdot \sigma^2$$

where Σ is the variance-covariance matrix. 4

(b) Define marginal distribution and conditional distribution of a p variate random variable. 2

20. *(a)* Obtain characteristic function of a p variate normal distribution $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. 4

(b) If $\underset{\sim}{X} \sim N(0, \Sigma)$, then prove that $X'\Sigma^{-1}X$ follows χ^2-distribution with 3 degrees of freedom. 2

21. *(a)* Show that

$$R^2_{1\cdot 23 \cdots n} = 1 - \frac{w}{w_{11}}$$

where the symbols have their usual notations. 4

(b) Given that

$$1 - R^2_{1\cdot 23} = (1 - r^2_{12})(1 - r^2_{13\cdot 2})$$

Deduce that $R_{1\cdot 23} \geq r_{12}$ and $R^2_{1\cdot 23} = r^2_{12} + r^2_{13}$, if $r_{23} = 0$. 2

22. Describe the sequential procedure for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where θ is the parameter of the Poisson distribution. Also obtain the ASN and OC functions. 6

23. Let X has the distribution

$$f(x, \theta) = \theta^x (1-\theta)^{1-x}; \quad x = 0, 1$$
$$0 < \theta < 1$$

for testing $H_0 : \theta = \theta_0$, against $H_1 : \theta = \theta_1$. Construct SPRT and obtain its OC function. 6

24. Explain Kolmogorov-Smirnov test for one sample. 6

25. Stating assumptions, explain Wilcoxon one-sample signed rank test. 6

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