2021/TDC/CBCS/ODD/
STSHCC-501T/117

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

STATISTICS

(5th Semester)

Course No. : STSHCC-501T

(Stochastic Process and Queuing Theory)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* from the following : 2×10=20

1. Define probability-generating function.

2. What are strong stationary process and wide sense stationary process?

3. Define discrete random sequence and discrete random process.

Define Markov process.

What is transition probability matrix (t.p.m.)?

Define Markov chain of order 1.

Define Poisson process.

Obtain moment-generating function of a Poisson process $\{X(t)\}$.

State the probability distribution of a Poisson process $\{X(t)\}$.

What are queue length and waiting time of a queuing system?

State the differential equations of $\{X(t)\}$ in a queuing model.

Write the steady state probabilities of a Poisson queue system in M/M/1 queuing model.

What is renewal process?

Define birth and death process.

Define the expected duration of game.

SECTION—B

Answer any *five* from the following : 6×5=30

16. Define stationary process. State and prove first-order stationary process. 3+3=6

17. If (X, Y) be a bivariate random variable, then find—

(a) marginal probability of X and Y;

(b) conditional expectation of $X \mid Y = j$;

(c) bivariate probability-generating function. 2+2+2=6

18. State Chapman-Kolmogorov equations. 6

19. The transition probability matrix (t.p.m.) of a Markov chain $\{X_n, n \geq 0\}$ with three states 0, 1, 2 is

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

and the initial distribution

$$P(X_0 = 1) = \frac{1}{3}, \quad i = 0, 1, 2$$

Find—

(a) $P[X_1 = 1 \mid X_0 = 2] = \frac{3}{4}$

(b) $P[X_2 = 2 \mid X_1 = 1] = \frac{1}{4}$

(c) $P[X_2 = 2, X_1 = 1 \mid X_0 = 2]$

(d) $P[X_2 = 2, X_1 = 1, X_0 = 2]$ 1+1+2+2=6

20. Prove that mean and variance of a Poisson process $\{X(t)\}$ are equal. 6

21. Derive the probability distribution of a Poisson process $\{X(t)\}$. 6

22. Derive the average number of customers in a queuing system of an M/M/1/N/FIFO model. 6

23. A one-person barber shop has six chairs to accommodate people waiting for haircut. Assume that the customers who arrive when all the six chairs are full, leave without entering the barber shop. Customers arrive at the shop one after another at an average of every 20 minutes and spend an average of 15 minutes in a barber's chair. Find—

(a) the probability that a customer can get directly into the barber's chair;

(b) the probability that there are at least three customers in the shop. 3+3=6

24. In a gambler's ruin problem, if f_i denotes the probability that starting with i amount of money, the gambler eventually reaches N. Show that

$$f_i = \frac{1-(q/p)^i}{1-(q/p)^N}, \quad \text{if } p \neq \frac{1}{2}$$
$$= \frac{i}{N}, \quad \text{if } p = \frac{1}{2}$$

where p and q are having usual meanings. 6

25. A gambler's luck follows a pattern. If he wins a game, the probability that he will win the next game is 0·6. However, if he loses a game, the probability of his losing the next game is 0·7. There is an even chance that the gambler wins the first game. What is the probability that he wins—

(a) the second game;

(b) the third game;

(c) in long-run? 2+2+2=6

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