# 2022/TDC/ODD/SEM/STSHCC–501T/117

## TDC (CBCS) Odd Semester Exam., 2022

## STATISTICS

( Honours )

**( 5th Semester )**

Course No. : STSHCC–501T

**( Stochastic Process and Queueing Theory )**

*Full Marks* : 50

*Pass Marks* : 20

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

UNIT—I

**1.** Answer any *two* of the following questions : 2×2=4

(a) Define probability generating function.

(b) What is bivariate probability generating function?

(c) Derive probability generating function (pgf) of Poisson distribution.

**2.** Answer any *one* of the following questions : 6

(a) Define Stochastic process. What are continuous random sequence and continuous random process? 2+2+2=6

(b) Define stationary process. Explain strongly stationary process and wide sense stationary process. 2+2+2=6

UNIT—II

**3.** Answer any *two* of the following questions : 2×2=4

(a) Define Markov chain.

(b) When is a Markov chain said to be homogeneous and non-homogeneous?

(c) Define state space. What are discrete and continuous state spaces?

**4.** Answer any *one* of the following questions : 6

(a) Consider a Markov chain with state space {0, 1} and transition probability matrix (t.p.m.)

$$\begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

Show that state 0 is persistent. What is irreducible Markov chain? 4+2=6





(b) (i) Draw a transition graph of the following Markov chain with transition probability matrix : 2

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \end{array}$$

(ii) A particle performs a random walk with absorbing barriers say as 0 and 4. Whenever it is at any position $r(0 < r < 4)$ it moves to $r+1$, with probability $p$ or to $(r-1)$ with probability $q$, $p+q=1$. But as soon as it reaches to 0 or 4, it remains there itself. Write down the Markov chain and the transition matrix. 2

(iii) Define transition probability matrix (t.p.m.). 2

UNIT—III

**5.** Answer any *two* of the following questions : 2×2=4

(a) Why is Poisson not a stationary process?

(b) Obtain the mean value of the Poisson process $\{X(t)\}$.

(c) A customer arrives at a bank counter in accordance with a Poisson process, with a mean rate of 2 per minute. Find the probability that during the time interval of 3 minutes, exactly four customers arrive.

6. Answer any *one* of the following questions : 6

(a) Prove that the interval time of a Poisson process $\{X(t)\}$ with occurrence of rate $\lambda$ has an exponential distribution with mean $1/\lambda$.

(b) If $\{X(t)\}$ and $\{Y(t)\}$ are two independent Poisson process, then show that the conditional distribution of $\{X(t)\}$ given $\{X(t)+Y(t)\}$ is binomial.

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Define arrival pattern of a queueing system.

(b) What is traffic intensity of a queueing system?

(c) Define waiting time of a customer in the queue.





8. Answer any *one* of the following questions : 6

(a) Obtain the differential equation of a queueing model $M/M/1/FIFO/N$.

(b) A departmental store has a single cashier. During the rush hour, customers arrive at a rate of 20 customers per hour. The cashier takes on an average 2·5 minutes per customer for processing.

(i) What is the probability that the cashier is idle?

(ii) What is the probability that a customer shall have to wait in a queue?

(iii) What is average number of customers in the queueing system?

(iv) What is the average time spent by a customer in the system?

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define birth and death process.

(b) If the game terminates when the gambler's capital becomes $O$ or $N$, then what is the transition probability of the Markov chain $\{X_n : n = 0, 1, 2, \cdots\}$?

(c) What is renewal process?

**10.** Answer any *one* of the following questions : 6

(a) Obtain the differential equations of birth and death process.

(b) If $B_i$ denote the duration of game, i.e., the number of bets, until the gambler's fortune reaches to $O$ or $N$, when gambler starts with the amount $i$, then find the expected duration of game.

★ ★ ★