2020/TDC (CBCS)/ODD/SEM/
STSHCC–501T/117

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

STATISTICS

(5th Semester)

Course No. : STSHCC–501T

(Stochastic Process and Queueing Theory)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Define generating function.

(b) What is bivariate generating function?

(c) Define stochastic process and state space.

(d) Define strict sense stationary process.

(e) When is a Markov chain said to be homogeneous and non-homogeneous?

(f) When are the states of a Markov chain said to be persistent and transient?

(g) Define transition probability matrix.

(h) What is random walk?

(i) State the postulates of Poisson process.

(j) Prove that the sum of two independent Poisson processes is a Poisson process.

(k) Why is the Poisson process not a stationary process?

(l) Evaluate the mgf of a Poisson process $\{X(t)\}$.

(m) Define queueing discipline.

(n) What is traffic intensity?

(o) Define transient state of a queueing model.

(p) Derive the average waiting time of a customer in the queue, if he/she has to wait in M/M/1/N/FCFS model.

(q) Define birth and death processes.

(r) State the differential equation of birth and death processes.

(s) If the game terminates when the gambler's capital become zero or N, then what is the transition probability of the Markov chain $\{X_n, n = 0, 1, 2, \cdots\}$?

(t) Define the expected duration of game.

SECTION—B

Answer *any* **five** questions

2. Explain 1st- and 2nd-order stationary processes. 3+3=6

3. Explain different classifications of random process. 6

4. (a) Draw the graph of the Markov chain $\{X_n, n \geq 0\}$ with the following transition probability matrix with states 1, 2, 3 : 3

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

(b) If $\{X_n, n \geq 0\}$ be a Markov chain having the state space $S = \{1, 2, 3, 4\}$ and transition probability matrix

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

then prove that—

(i) state '1' is persistent;

(ii) state '3' is transient. 3

5. (a) Define Markov chain. 2

(b) Write Chapman-Kolmogorov equation. 4

6. Find the probability distribution of Poisson process $\{X(t)\}$ and hence find its mean. 4+2=6

7. (a) Prove that Poisson process is a Markov process. 3

(b) If $\{X(t)\}$ and $\{Y(t)\}$ be two independent Poisson processes, then show that the conditional distribution of $\{X(t)\}$ given $\{X(t) + Y(t)\}$ is binomial. 3

8. (a) Obtain steady-state probability in M/M/1 model with finite system capacity and hence obtain the waiting time. 4

(b) Obtain average number of customers in the system in M/M/1 model with finite system capacity. 2

9. Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains, find—

(a) the probability that the yard is empty;

(b) the average number of trains in the system. 6

10. In a gambler's ruin problem, if f_i denotes the probability that starting with i amount of money the gambler eventually reaches N, then show that

$$f_i = \frac{1 - \left(\frac{q}{p}\right)^i}{1 - \left(\frac{q}{p}\right)^N}, \quad \text{if } p \neq \frac{1}{2}$$

$$= \frac{i}{N}, \quad p = \frac{1}{2}$$

with p and q are having usual meanings. 6

11. A gambler with capital z plays against an adversary with capital $(a-z)$. The game is played in stages. At each stage, the gambler can win one unit with probability p and loss one unit with probability q, $p+q=1$. Derive the expression for ultimate win of the gambler. 6

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