2023/TDC(CBCS)/ODD/SEM/
STSHCC–501T/117

TDC (CBCS) Odd Semester Exam., 2023

STATISTICS

(Honours)

(5th Semester)

Course No. : STSHCC–501T

(Stochastic Process and Queueing Theory)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Derive the p.g.f. of the geometric distribution.

2. Derive probability-generating function of binomial distribution.

3. Give two examples of stochastic process.

UNIT—II

4. Define transition probability matrix.

5. Define communicative state. Write the properties of the communicative state of a Markov chain.

6. Define persistent and recurrent states of a Markov chain.

UNIT—III

7. State the postulates of a Poisson process.

8. State and prove the additive property of a Poisson process.

9. Define branching process with example.

UNIT—IV

10. What do you mean by steady-state distribution?

11. Define single channel and multichannel queuing system.

12. Define waiting time distribution. How do you calculate waiting time in a Poisson distribution?

(Continued)

UNIT—V

13. Give an example of Gambler's ruin problem.

14. What is a classical ruin problem?

15. What is the risk of ruin in gambling?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. Find the mean and variance of Bernoulli distribution, Poisson distribution and binomial distribution using p.g.f. 2+2+2=6

17. Define the following terms : 2×3=6

(a) Mean stationary process

(b) Variance stationary process

(c) Covariance stationary process

UNIT—II

18. Discuss higher order transition probability of a Markov chain on the basis of Chapman-Kolmogorov equation. 6

(Turn Over)

19. Consider a Markov chain of states $S = (0, 1, 2)$ and transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \end{pmatrix} \end{matrix}$$

Verify if the chain is irreducible or not. 6

UNIT—III

20. Derive the probability function of the Poisson process. 6

21. Show that a random selection from a Poisson process is also a Poisson process. 6

UNIT—IV

22. Suppose computer programs are submitted for execution on a university's central computing facility and that these programs arrive at a rate of 10 per minute. Assume average run-time for a program is five seconds and that both interarrival times and run-times are exponentially distributed. During what fraction of the time is the CPU idle? What is the number of jobs in the job queue?

(Continued)

23. Consider a typical barber shop. Demonstrate that it is queueing system by describing its components.

UNIT—V

24. Obtain the equations for birth and death processes.

25. Consider a Gambler who at each play of the game has probability p of winning one unit and probability $q = 1 - p$ of losing one unit. Assuming that successive plays of the game are independent, what is the probability that; starting with i units, the Gambler's fortune will reach N before reaching 0?

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