**2023/TDC (CBCS)/EVEN/SEM/**
**STSHCC–401T/270**

**TDC (CBCS) Even Semester Exam., 2023**

# STATISTICS

( Honours )

**( 4th Semester )**

Course No. : STSHCC–401T

**( Statistical Inference )**

*Full Marks* : 50
*Pass Marks* : 20

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## SECTION—A

Answer any *ten* of the following questions : 2×10=20

**1.** Define consistency with an example.

**2.** What do you mean by estimation? Give an example.

**3.** Define sufficiency with an example.

**4.** What is method of moment estimation? Give an example.

5. Explain invariance property of maximum likelihood estimator.

6. Give an example to show that both method of moment and method of maximum likelihood provide the same estimate of the parameter.

7. What do you mean by test of significance?

8. Define null and alternative hypothesis with a suitable example.

9. Explain the difference between simple hypothesis and composite hypothesis with an example.

10. Write the statement of Neyman-Pearson lemma and provide a purpose of the lemma.

11. What are the basic differences between Neyman-Pearson lemma and likelihood ratio test?

12. What are the properties of likelihood ratio test?





13. What do you mean by interval estimation? Give an example of interval estimation.

14. Give an example of pivotal quantity in confidence interval.

15. What do you understand by large sample confidence intervals?

## SECTION—B

Answer any *five* of the following questions : 6×5=30

16. Define minimum variance unbiased estimator and minimum variance bound estimator and explain clearly the difference between them.

17. Provide an example of an estimator *(a)* which is consistent but not unbiased and *(b)* which is unbiased but not consistent.

18. If a sufficient estimator exists, show that it is a function of maximum likelihood estimator.

19. Let $x_1, x_2, ..., x_n$ denote random sample of size $n$ from a uniform population with p.d.f.

$$f(x, \theta) = 1; \quad \theta - \frac{1}{2} \leq x \leq \theta + \frac{1}{2}, \quad -\infty < \theta < \infty$$

Obtain MLE of $\theta$.

**20.** Let $X$ has a probability density function of the form

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta};\ 0 < x < \infty,\ \theta > 0$$

To test $H_0 : \theta = 2$ against $H_1 : \theta = 1$, use the random sample $x_1$, $x_2$ of size 2 and define a critical region

$$W = \{(x_1, x_2) : 9 \cdot 5 \le x_1 + x_2\}$$

Find—

*(a)* power of the test;

*(b)* significance level of the test.

**21.** Derive a most powerful test of the hypothesis $H_0 : \theta = \frac{1}{4}$ against $H_1 : \theta = \frac{1}{2}$ for the parameter $\theta$ in a geometric distribution having probability mass function

$$P(x) = \theta(1-\theta)^x;\ \ x = 0, 1, 2, \ldots$$

based on a random sample of size 2.

**22.** Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size $n$ from a Bernoulli distribution having probability mass function

$$P(x) = \theta^x (1-\theta)^{1-x};\ \ x = 0, 1, \ldots$$

Obtain a uniformly most powerful size $\alpha$ test for $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. Would you modify the test if $H_1 : \theta < \theta_0$?



( Continued )

**23.** On the basis of a single observation $x$ from the following probability density function

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta} : x > \theta,\ \ \theta > 0$$

the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 4$ is tested by using a set $C = \{x : x > 3\}$ as the critical region. Prove that a critical region $C$ provides a most powerful test of its size. What is the power of the test?

**24.** Develop a general method for obtaining confidence intervals. Obtain a $100(1-\alpha)\%$ confidence interval for large sample size for the parameter $\theta$ of the Poisson distribution.

**25.** Explain pivotal quantity method of constructing confidence interval with an example.

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