2022/TDC (CBCS)/EVEN/SEM/
STSHCC–401T/126

TDC (CBCS) Even Semester Exam., 2022

STATISTICS

(Honours)

(4th Semester)

Course No. : STSHCC–401T

(Statistical Inference)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. State Cramer-Rao inequality.

2. What is unbiasedness? Give an example.

3. What is a statistic? When the statistic is sufficient?

4. (a) Sample mean is an unbiased estimate of ____. (Fill in the blank)

   (b) Name a method of estimation.

5. If $X \sim$ Poisson (μ), then find the maximum likelihood estimator of μ.

6. Write two properties of the estimators obtained by the method of moments.

7. If $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, then what is the estimator of λ using the method of moments?

8. Define *(i)* null hypothesis and *(ii)* alternative hypothesis.

9. Explain the concept of best critical region.

10. Explain the concept of level of significance.

11. Out of type–I and type–II errors, which one is more serious and why?

12. Out of the following, which is a simple and which is a composite hypothesis related to a normal distribution where both μ and σ^2 is unknown?

    (a) $\mu = \mu_0$ (specified)

    (b) $\mu = 5,\ \sigma^2 = 3$

    (c) $\sigma^2 = \sigma_0^2$ (specified)

    (d) $\mu > 5,\ \sigma^2 = 3$

    (Choose the correct one)

13. State Neyman-Pearson lemma. Also write one purpose of the lemma.

14. What do you understand by confidence region?

15. What is the shortest length confidence interval?

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. Deduce $1-\alpha$ level confidence interval for the binomial proportion.

17. Let $x_1, x_2, \ldots, x_n$ be a random sample from an exponential distribution

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \ \theta > 0$$
$$= 0, \qquad \text{otherwise}$$

Compute $1-\alpha$ level confidence interval for θ.

18. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Bin(1, p) random variables and let $H_0 : p = p_0$, $H_1 : p = p_1$ $(p_1 > p_0)$. Use Neyman-Pearson lemma to find the most powerful test of size α for testing H_0 against H_1.

19. Define most powerful test. State the properties of likelihood ratio test.

20.
$$f(x, \theta) = \frac{1}{\theta}, \quad 0 \le x \le \theta$$
$$= 0, \quad \text{otherwise}$$

$H_0 : \theta = 1$ against $H_1 : \theta = 2$, the sample size is one. Find out type–I and type–II errors if the critical region is *(a)* $x \ge 0 \cdot 5$ and *(b)* $x \ge 1$.

21. What do you mean by power of a test? What is the ideal relation between the size of a test and power of a test?

22. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Bin(n, p) random variables where both n and p are unknown. Obtain their estimates by using method of moments.

23. Show, with the help of an example, that the MLE is not necessarily unbiased.

24. State and prove the invariance property of a consistent estimator.

25. Explain the concept of 'consistency' and 'unbiasedness' of estimators. Give an example of each case.

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