2020/TDC(CBCS)/ODD/SEM/
STSHCC-303T/114

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

STATISTICS

(3rd Semester)

Course No. : STSHCC-303T

(Mathematical Analysis)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Why does the set *R* of real numbers is a complete ordered field?

(b) Give one example respectively for bounded set and unbounded set.

(c) What is the concept of neighbourhood of a point?

(d) Define open set and closed set.

(e) What do you mean by an infinite series?

(f) Mention the necessary condition for convergence of a infinite series.

(g) Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent.

(h) Explain comparism test of two positive term series.

(i) State mean-value theorem for derivatives.

(j) What is Maclaurin's infinite series expansion of a function $f(x)$ in powers of x?

(k) Write series expansion of the function $\log(1+x)$.

(l) Examine the validity of the hypothesis and conclusion of Lagrange's mean-value theorem of the following :

$$f(x)=x(x-1)(x-2) \text{ on } \left[0, \frac{1}{2}\right]$$

(m) Find the value of $\Delta \tan^{-1} x$.

(n) Explain the use of interpolation.

(o) If $f(x)=\frac{1}{x^2}$, find the value of its 2nd divided difference with arguments a, b, c.

(p) Evaluate $(\Delta^2 e^x)/(\Delta e^x)^2$.

(q) Show that

$$\mu=\sqrt{1+\frac{1}{4}\delta^2}$$

where symbols have their usual meanings.

(r) Define difference equation of first order.

(s) What is meant by numerical integration?

(t) Mention the assumptions of Simpson's one-third rule.

SECTION—B

Answer *any* **five** questions

2. *(a)* Define a real sequence with example. Prove that a sequence cannot converge to more than one limit. 2+2=4

(b) State Cauchy's first theorem on limit. 2

3. (a) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{(n-1)!}$$

is convergent. 3

(b) Explain limit superior and limit inferior of a bounded sequence. 3

4. (a) Explain d'Alembert's ratio test for positive-term series. 3

(b) Test for convergence of the series

$$1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \cdots$$

for $x > 0$. 3

5. (a) What do you mean by absolute convergence of series? 2

(b) Define an alternating series and discuss the Leibniz's test for convergence of such series. Illustrate your answer with an example. 4

6. State and prove Lagrange's mean-value theorem. Hence give interpretation of this theorem. 4+2=6

7. (a) Explain Taylor's theorem with Lagrange's form of remainder. 3

(b) Obtain Maclaurin's series expansion of the function $\sin x$. 3

8. State and prove Newton's backward interpolation formula and mention the criteria for its use. 5+1=6

9. (a) Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$. 3

(b) If $U_0 = a$, $U_2 = b$, $U_4 = c$, prove that

$$U_x = a + \frac{x}{2}(b-a) + \frac{x(x-2)}{8}(c - 2b + a)$$ 3

10. (a) Derive Simpson's one-third rule. 3

(b) If $u_x = a + bx + cx^2$, prove that

$$\int_1^3 u_x dx = 2u_2 + \frac{1}{12}[u_0 - 2u_2 + u_4]$$

Hence find

$$\int_{-1/2}^{1/2} e^{-\frac{x^2}{10}} dx$$ 3

11. (a) Solve : 2

$$u_{x+1} - 3^x u_x = 0, \ x > 0$$

(b) Obtain Stirling's interpolation formula. 4

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