2021/TDC/CBCS/ODD/
STSHCC-303T/114

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

STATISTICS

(3rd Semester)

Course No. : STSHCC–303T

(Mathematical Analysis)

Full Marks : 50
Pass Marks : 20
Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Define a real sequence with an example.

2. What do you mean by complete ordered field?

3. Define open set and closed set.

4. Define positive term series with an example.

5. How can you conclude that a given series is convergent or not?

6. Is the series $\sum \frac{1}{n}$ convergent?

7. Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ on $[2, 5]$.

8. Write series expansion of $\cos x$.

9. State mean value theorem for derivatives.

10. Find the value of $\frac{\Delta^2}{E} x^3$.

11. Prove that $\Delta\, {}^nC_{x+1} = {}^nC_x$, where Δ operates over n only.

12. Prove that the third divided difference with arguments a, b, c, d of $f(x) = \frac{1}{x}$ is $-\frac{1}{abcd}$.

13. Define the operators δ and μ.

14. Obtain relation between operators E and δ.

15. What do you mean by numerical integration?

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. (a) Prove that a sequence cannot converge to more than one limit. 2

(b) State and prove Cauchy's first theorem on limits. 4

17. (a) Show that the sequence $\{S_n\}$, where

$$S_n = \left(1 + \frac{1}{n}\right)^n$$

is convergent and limit of S_n lies between 2 and 3. 3

(b) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$$

cannot converge. 3

18. (a) Explain Cauchy's nth root test. 2

(b) What is meant by absolute convergence of series? 2

(c) Test the convergence of the series

$$\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots, \; x > 0$$ 2

19. *(a)* Discuss about Lebnitz's test for convergence of alternating series. 2

(b) Explain D'Alembert's ratio test. 2

(c) Test for convergence of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, \quad x>0$$ 2

20. *(a)* State and prove Rolle's mean value theorem. 4

(b) Give geometrical interpretation of Lagrange's mean value theorem. 2

21. *(a)* Explain Taylor's theorem with Lagrange's form of remainder. 3

(b) What is Taylor's series? 1

(c) Obtain Maclaurin's series expansion of $(1+x)^n$, when n is a positive integer. 2

22. *(a)* State Lagrange's interpolation formula. If $u_0 = a$, $u_2 = b$ and $u_4 = c$, prove that

$$u_x = a + \frac{x}{2}(b-a) + \frac{x(x-2)}{8}(c-2b+a)$$ 1+3=4

(b) Mention the assumptions of using interpolation technique. 2

23. State and prove Newton's forward interpolation formula and mention the conditions for its use. 5+1=6

24. *(a)* Derive Simpson's three-eighths rule. 4

(b) If $f(x)$ can be represented by a polynomial of degree 2, then prove that

$$\int_0^1 f(x)dx = \frac{1}{12}[5f(0)+8f(1)-f(2)]$$ 2

25. *(a)* Obtain Gauss forward interpolation formula. 4

(b) Solve : 2

$$u_{x+1} - \frac{1}{x}u_x = 0, \quad x>0$$

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