2019/TDC/ODD/SEM/STSHCC–303T/119

TDC (CBCS) Odd Semester Exam., 2019

STATISTICS

(3rd Semester)

Course No. : STSHCC–303T

(Mathematical Analysis)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Show that every infinite bounded set has a limit point.

(b) What is closed set? Prove that a set is closed if its complement is open.

(c) Define limit superior and limit inferior of a bounded sequence.

(Turn Over)

Answer *either* Q. No. **2** *or* Q. No. **3** :

2. *(a)* State Cauchy's general principle of convergence. Hence test the convergence of sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{4} + \frac{1}{7} + \cdots + \frac{1}{3n-2}$$ 2+2=4

(b) Show that

$$\lim \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2} \right] = 0$$ 2

3. *(a)* State and prove Cauchy's first theorem on limits. 4

(b) What do you mean by monotonic sequences? What happens to a monotonic increasing sequence if it is not bounded above? 2

UNIT—II

4. Answer any *two* of the following : 2×2=4

(a) State the necessary condition for convergence of an infinite series.

(b) What is an alternating series?

(c) For the series

$$\frac{1}{1^p} + \frac{1}{2^p} + \cdots + \frac{1}{n^p} + \cdots$$

give the condition under which series is *(i)* convergent and *(ii)* divergent.

Answer *either* Q. No. **5** *or* Q. No. **6** :

5. *(a)* Explain comparison test for positive term series. 3

(b) Test for convergence of the series

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \cdots + \frac{(n+1)x^n}{n^3} + \cdots\cdots$$ 3

6. *(a)* Discuss about Leibnitz's test for convergence of alternating series with an example. 3

(b) Test the convergence of the series

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots\cdots$$ 3

UNIT—III

7. Answer any *two* of the following : 2×2=4

(a) Verify Rolle's mean value theorem in the function $f(x) = x^3 - 4x$ on $[-2, 2]$.

(b) Give geometrical interpretation of Lagrange's mean value theorem.

(c) Expand $\log(1+x)$ by using Maclaurin's theorem.

Answer *either* Q. No. **8** *or* Q. No. **9** :

8. (a) State Taylor's theorem with Lagrange's form of remainder. 2

(b) State and prove Lagrange's mean value theorem. 4

9. (a) Obtain Maclaurin's infinite series expansion of a function $f(x)$ in powers of x. 3

(b) Expand $(1+x)^n$ by using Maclaurin's infinite series expansion. 3

UNIT—IV

10. Answer any *two* of the following : 2×2=4

(a) Prove that

$$u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_3$$

(b) Prove that

$$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

(c) Obtain third divided difference of the function $\frac{1}{x}$ with arguments a, b, c, d.

Answer *either* Q. No. **11** *or* Q. No. **12** :

11. State and prove Newton's divided difference formula and show that it is a particular case of Newton's forward formula. 4+2=6

12. (a) State and prove Newton's forward interpolation formula. 3

(b) By means of Lagrange's formula, prove that

$$u_0 = \frac{1}{2}(u_1 + u_{-1}) - \frac{1}{8}\left[\frac{1}{2}(u_3 - u_1) - \frac{1}{2}(u_{-1} - u_{-3})\right] \quad 3$$

UNIT—V

13. Answer any *two* of the following : 2×2=4

(a) Prove that

$$\delta = \Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$$

(b) Find the value of

$$\delta^{-1} = \frac{E^{\frac{1}{2}}}{E-1}$$

where the symbols have their usual meanings.

Answer *either* Q. No. **14** *or* Q. No. **15** :

14. *(a)* Obtain general quadrature formula of numerical integration. 3

(b) If the third differences are constant, prove that

$$\int_0^2 U_x\,dx = \frac{1}{24}\left[U_{-\frac{1}{2}} + 23\left(U_{\frac{1}{2}} + U_{\frac{3}{2}}\right) + U_{\frac{5}{2}}\right]$$ 3

15. *(a)* Obtain Gauss forward interpolation formula with $(2n+1)$ equidistant arguments. 4

(b) What is Stirling's approximation to factorial n? 2

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