2023/TDC(CBCS)/ODD/SEM/
STSHCC–303T/114

TDC (CBCS) Odd Semester Exam., 2023

STATISTICS

(Honours)

(3rd Semester)

Course No. : STSHCC–303T

(Mathematical Analysis)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. What is closed set? Prove that a set is closed if its complement is open.

2. Show that every infinite bounded set has a limit point.

3. Define neighbourhood of a point. What is limit point of a set?

UNIT—II

4. Show that the series

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \cdots + \frac{1}{(\log n)^p} + \cdots$$

diverges for $p > 0$.

5. State Raabe's test.

6. What do you mean by an infinite series? Also state the condition necessary and sufficient for an infinite series to be convergent.

UNIT—III

7. State Rolle's theorem.

8. Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem of the following :

$$f(x) = 2x^2 - 7x + 10 \quad \text{on} \quad [2, 5]$$

9. Give the geometrical interpretation of Lagrange's mean value theorem.

UNIT—IV

10. Prove that

$$\Delta u_x v_x = u_{x+1} \cdot \Delta v_x + v_x \cdot \Delta u_x$$

11. Define E. Evaluate $(Ex^2)^2$.

12. Prove that

$$e^x = \frac{\Delta^2}{E} \cdot e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$$

UNIT—V

13. State Gauss' forward and backward interpolation formula.

14. If $u_x = a + bx + cx^2$, then prove that

$$\int_1^3 u_x dx = 2u_2 + \frac{1}{12}(u_0 - 2u_2 + u_4)$$

15. What is numerical integration?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. *(a)* Prove that a sequence cannot converge to more than one limit. 2

(b) Show that the sequence $\{S_n\}$, where

$$S_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}$$

converges to zero. 4

17. *(a)* Show that

$$\lim \frac{(3n+1)(n-2)}{n(n+3)} = 3$$ 2

(b) State and prove Cauchy's first theorem on limits. 4

UNIT—II

18. *(a)* Explain D'Alembert's ratio test for positive term series. 3

(b) Test for convergence of the series

$$\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \frac{4^2 \cdot 5^2}{4!} + \cdots$$ 3

19. *(a)* What do you mean by absolute convergence of series? 2

(b) Test for the absolute convergence of the following series : 4

(i) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

(ii) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

UNIT—III

20. *(a)* Show that the number θ which occurs in the Taylor's theorem with Lagrange's form of remainder after n terms approaches the limit $\frac{1}{n+1}$ as $h \to 0$, provided that $f^{n+1}(x)$ is continuous and different from zero at $x = a$. 4

(b) Obtain Maclaurin's series expansion of $(1+x)^n$, when n is a positive integer. 2

21. *(a)* State and prove Lagrange's mean value theorem. 4

(b) What is Taylor's series? Write the Taylor's series expansion of $\sin x$ at $x = 0$. 2

UNIT—IV

22. (a) Prove that the nth difference of a polynomial of degree n is constant, when the values of the argument are equidistant. 5

(b) State the relation between ordinary difference and divided difference. 1

23. (a) Derive Lagrange's interpolation formula. 3

(b) Prove by Lagrange's formula

$$u_1 = u_3 - 0\cdot3(u_5 - u_{-3}) + 0\cdot2(u_{-3} - u_{-5})$$ 3

UNIT—V

24. State and prove Simpson's $\frac{1}{3}$rd rule. 6

25. Evaluate

$$\int_0^2 u_x dx$$

given

x	u_x
0	u_0
1	u_1
2	u_2

6

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