2022/TDC/ODD/SEM/STSHCC-301T/112

TDC (CBCS) Odd Semester Exam., 2022

STATISTICS

(Honours)

(3rd Semester)

Course No. : STSHCC–301T

(Limit Laws, Testing of Hypothesis and Sampling Distribution)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Define convergency in mean square.

(b) State Chebyshev's inequality.

(c) If X_n converges to X in probability, then prove that $X_n - X$ converges to zero in probability.

2. Answer any *one* of the following questions : 6

(a) (i) State and prove weak laws of large numbers. 3

(ii) An unbiased die is thrown 600 times. Use Chebyshev's inequality to find the lower bound for the probability of getting 80 to 120 sixes. 3

(b) (i) State strong laws of large numbers. 2

(ii) Examine if the law of large numbers holds for the sequence of independent random variables $\{X_n\}$ with the distribution of X_n, given by

$$f_n(x) = \frac{1}{|x|^3}, \quad |x| > 1$$
$$= 0, \quad \text{otherwise}$$
4

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define parameter and statistic.

(b) Prove that sample proportion is an unbiased estimate of population proportion.

(c) State p.d.f. of the *k*th order statistic.

4. Answer any *one* of the following questions : 6

(a) Obtain the sampling distribution of mean of a random sample, drawn from a (i) normal population and (ii) non-normal population. 3+3=6

(b) Obtain the distribution of the sample range R of a sample of size n, drawn from a rectangular population

$$dF = \frac{dx}{\theta}, \quad 0 \le x \le \theta$$

Also find $E(R)$ and $V(R)$.

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Define producer's risk and consumer's risk.

(b) What are critical region and level of significance?

(c) Write the steps for testing of hypothesis.

6. Answer any *one* of the following questions : 6

(a) (i) Describe the large sample test for single mean. 3

(ii) A die is thrown 9000 times and a throw of 1 and 2 is observed 3120 times. Show that the die cannot be regarded as an unbiased one. Also find the limits between which the probability of a throw of 1or 2 lies. 3

(b) (i) Describe the procedure for testing of standard deviation for two distinct populations. 2

(ii) Explain the utility of standard error in test of significance for large samples. 2

(iii) Define fiducial limit. 1

(iv) You are to carry out an investigation to find whether there is any difference in weight at birth of boys and girls. In this case, what is the appropriate null hypothesis? 1

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) State the conditions of validity of χ^2-test.

(b) If $\chi^2 \sim \chi^2_{(n)}$, then obtain mean and variance of χ^2-distribution.

(c) If $X_i \sim \chi^2_{(n_i)}$, $i = 1, 2, \ldots, k$, then prove that

$$\sum_{i=1}^{k} X_i \sim \chi^2$$

distribution with $\sum_{i=1}^{k} n_i$ degrees of freedom.

8. Answer any *one* of the following questions : 6

(a) If X_1 and X_2 are independent χ^2 variates with n_1 and n_2 degrees of freedom respectively, then prove that

$$U = \frac{X_1}{X_1 + X_2}$$

and $V = X_1 + X_2$ are independently distributed with U as a $\beta\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate and V as a χ^2-variate with $(n_1 + n_2)$ degrees of freedom.

(b) *(i)* Define χ^2-variate with n degrees of freedom. 1

(ii) Derive χ^2-distribution. 2

(iii) If X is a χ^2-variate with n degrees of freedom, then prove that for large n $\sqrt{2X} \sim N(\sqrt{2n}, 1)$. 3

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Write the p.d.f. of Student's *t*-distribution with n degrees of freedom. What are the fiducial limits of *t*-distribution at α% level of significance?

(b) State the assumptions of Student's *t*-statistic.

(c) Define *F*-statistic. Write the p.d.f. of Snedecor's *F*-distribution.

10. Answer any *one* of the following questions : 6

(a) If $t \sim t_{(n)}$ [Student's *t*-distribution with n degrees of freedom], then derive Student's *t*-distribution with n degrees of freedom.

(b) If F be the Snedecor's *F*-statistic and $F \sim F(\gamma_1, \gamma_2)$, then obtain the mean and variance of Snedecor's *F*-distribution.

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