2020/TDC(CBCS)/ODD/SEM/ STSHCC-301T/112

TDC (CBCS) Odd Semester Exam., 2020 held in March, 2021

STATISTICS

(3rd Semester)

Course No. : STSHCC-301T

(Limit Laws, Testing of Hypothesis and Sampling Distribution)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Define convergence in mean square. 2

(b) State strong law and weak law of large numbers. 1+1=2

(c) State Lyapunoff's central limit theorem. 2

(d) Define convergence in distribution. 2

(e) Explain the concept of standard error. What is the standard error of a sample mean? 1+1=2

(f) Define critical region and level of significance. 1+1=2

(g) Define sample range and sample median. 1+1=2

(h) Explain briefly the applications of order statistics. 2

(i) Obtain the c.d.f. and p.d.f. of first-order statistic $X_{(1)}$. 1+1=2

(j) Define simple and composite hypotheses with examples. 1+1=2

(k) Define critical value. What is statistical hypothesis? 1+1=2

(l) Write notes on one-tailed and two-tailed tests. 1+1=2

(m) Define chi-square variate and write its p.d.f. for n-degrees of freedom. 1+1=2

(n) State the applications of chi-square distributions. 2

(o) State and prove the additive property of chi-square variates. 2

(p) Obtain the m.g.f. of χ^2-distribution with n-degrees of freedom. 2

(q) Write down the applications of t-distribution. 2

(r) Define Snedecor's F-statistic and write its p.d.f. 1+1=2

(s) Write the applications of F-distribution. 2

(t) Discuss how to test the significance of an observed correlation coefficient r, when population correlation coefficient $\rho = 0$. 2

SECTION—B

Answer *any* **five** questions

2. (a) If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, use Chebyshev's inequality to determine a lower bound for the probability $P[-2 < X < 8]$. 3

(b) State and prove de Moivre-Laplace central limit theorem. 3

3. (a) Let $X_1, X_2, \cdots, X_n$ be i.i.d. Poisson variates with parameter λ. Use central limit theorem (CLT) to estimate

$$P[120 \le S_n \le 160]$$

where $S_n = X_1 + X_2 + \cdots + X_n$; $\lambda = 2$ and $n = 75$. 3

(b) State central limit theorem. Also explain some applications of central limit theorem. 1+2=3

4. (a) Obtain the joint p.d.f. of r-th and s-th order statistics $X_{(r)}$ and $Y_{(s)}$. 3

(b) Derive the distribution of sample median. 3

5. (a) Obtain the sampling distribution of mean of a random sample drawn from a normal population. 3

(b) Show that the standard error of sample mean of a sample of size n drawn without replacement is

$$\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

where N = population size

σ^2 = population variance 3

6. (a) Describe the large sample test for difference of means of two distinct populations. 3

(b) Explain the concept of p-value. Obtain the test of significance for large samples for single mean. 1+2=3

7. (a) Describe the test of significance for large samples for difference between two proportions. 3

(b) Obtain the large sample test for difference of standard deviations for two distinct populations. 3

8. (a) If χ_1^2 and χ_2^2 are two independent χ^2-variates with n_1 and n_2 degrees of freedom, then prove that

$$\frac{\chi_1^2}{\chi_2^2} \text{ is a } \beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$$ 3

(b) Obtain mode and skewness of χ^2-distribution with n-degrees of freedom. 3

9. (a) If

$$X \sim \chi^2_{(n)}$$

then prove that

$$\frac{X-n}{\sqrt{2n}} \sim N(0,1)$$

where symbols have their usual meanings. 3

(b) For a 2×2 contingency table

a	b
c	d

prove that χ^2-test of independence of attribute gives

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

where $N = a+b+c+d$. 3

10. (a) Describe the test of significance of mean of a univariate normal population in case of small sample and hence obtain the confidence interval of population mean. 3

(b) For the t-distribution with n.d.f., establish the recurrence relation

$$\mu_{2r} = \frac{n(2r-1)}{(n-2r)} \cdot \mu_{2r-2},\ n > 2r$$ 3

11. (a) Obtain the relation between t-distribution and F-distribution and also between χ^2-distribution and F-distribution. 3

(b) Derive the distribution of Snedecor's F-statistic. 3

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