2019/TDC/ODD/SEM/
STSHCC–301T/117

TDC (CBCS) Odd Semester Exam., 2019

STATISTICS

(3rd Semester)

Course No. : STSHCC–301T

(Limit Laws, Testing of Hypothesis and Sampling Distribution)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following questions : 2×2=4

(a) State strong and weak law of large numbers.

(b) Define convergence in probability.

(c) Define central limit theorem.

Answer *either* Q. No. **2** *or* Q. No. **3** :

2. *(a)* State and prove Chebyshev's inequality.

(b) Examine if the law of large numbers holds for the sequence of independent random variables $\{X_n\}$ with the distribution of X_n given by

$$f_n(x) = \frac{1}{|x|^3}, \qquad |x| > 1$$

$$= 0, \qquad \text{otherwise}$$

3

3. *(a)* State and prove Bernoulli's law of large numbers. 3

(b) State and prove de Moivre-Laplace theorem. 3

UNIT—II

4. Answer any *two* from the following questions : 2×2=4

(a) Define sampling distribution of a statistic.

(b) State the cumulative distribution function and the probability density function of first-order statistics.

(c) Write the standard error (SE) of sample variance.

Answer *either* Q. No. **5** *or* Q. No. **6** :

5. *(a)* Find the distribution of first-order statistics for an exponential distribution with mean $\frac{1}{\theta}$. 3

(b) Show that for a random sample of size 2 from $N(0, \sigma^2)$ population $E[X_{(1)}] = -\sigma / \sqrt{\pi}$. 3

6. *(a)* Derive the density function of r-th order statistics. 3

(b) Obtain the distribution of the sample range R in a sample of size n from the rectangular population $U(0, \theta)$. 3

UNIT—III

7. Answer any *two* from the following questions : 2×2=4

(a) Define producer's risk and consumer's risk.

(b) Define critical value and level of significance.

(c) Write the uses of standard error in testing of significance for large samples.

Answer *either* Q. No. **8** *or* Q. No. **9** :

8. *(a)* What is statistical hypothesis? Explain two types of errors, arise in testing a hypothesis. 1+2=3

(b) Explain the role played by the central limit theorem in large sample theory of testing of hypothesis. 3

9. *(a)* Obtain the large sample test for difference of proportions of two distinct populations. 3

(b) Describe the test of significance for large samples to test the difference of means of two distinct populations. 3

UNIT—IV

10. Answer any *two* from the following questions : 2×2=4

(a) Define χ^2-statistic and state the characteristic function of χ^2-distribution.

(b) Write the p.d.f. of χ^2-distribution and state the relationship between the mean and variance of χ^2-distribution.

(c) State the conditions of validity of χ^2-test.

Answer *either* Q. No. **11** *or* Q. No. **12** :

11. *(a)* Derive the cumulant generating function (CGF) of χ^2-distribution. Hence find the mean and variance of χ^2-distribution. 3

(b) If χ_1^2 and χ_2^2 are independent variates with n_1 and n_2 degrees of freedom, and

$$u = \frac{\chi_1^2}{\chi_1^2 + \chi_2^2}$$

$$v = \chi_1^2 + \chi_2^2$$

are independently distributed, then prove that—

$$\frac{\chi_1^2}{\chi_1^2 + \chi_2^2} \sim \beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$$

and $\chi_1^2 + \chi_2^2 \sim \chi^2(n_1 + n_2)$. 3

12. *(a)* If X_i follows $\chi^2_{(n_i)}, i = 1, 2, \cdots, k,$ then prove that $\sum_{i=1}^{k} X_i$ follows χ^2 distribution with $\sum_{i=1}^{k} n_i$ degrees of freedom. 3

(b) If X is a χ^2 variate with n degrees of freedom, then prove that for large n

$$\sqrt{2X} \sim N(\sqrt{2n}, 1)$$ 3

UNIT—V

13. Answer any *two* from the following questions : 2×2=4

(a) Define Snedecor's F-statistic and write the probability density function F-statistic.

(b) Write the test statistic for testing the significance of an observed sample correlation coefficient.

(c) Show that for Student's t-distribution with n degrees of freedom, mean deviation about mean is given by

$$\frac{\sqrt{n}}{\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$$

Answer *either* Q. No. **14** *or* Q. No. **15** :

14. (a) If $x \sim t_{(n)}$, then prove that $E(X) = 0$ and

$$V(X) = \frac{n}{n-2}, \; n > 2.$$ 3

(b) Show how probability points of $F(n_2, n_1)$ can be obtained from those of $F(n_1, n_2)$. 3

15. (a) Derive the relation between t and F. 3

(b) Prove that if $n_1 = n_2$, the median of F-distribution is at $F = 1$ and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1 Q_3 = 1$. 3

★ ★ ★