TDC (CBCS) Odd Semester Exam., 2023

STATISTICS

(Honours)

(3rd Semester)

Course No. : STSHCC–301T

(Sampling Distributions)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

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SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Define convergence in distribution.

2. State weak law of large numbers.

3. If X_n converges to X in probability, then prove that $X_n - X$ converges to zero in probability.

UNIT—II

4. Obtain the cumulative distribution function of rth order statistics.

5. If $x_1, x_2, \cdots, x_n$ be a random sample of size n having sample mean $\bar{x}$, drawn from a population in SRSWR having variance σ^2, then find variance of sample mean.

6. Define random sample and sampling distribution of a statistic.

UNIT—III

7. Define simple and composite hypotheses with example.

8. Define type-I and type-II errors.

9. Obtain standard error of sample proportion.

UNIT—IV

10. Define χ^2 variate and write its p.d.f. with n degrees of freedom.

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11. Let $X \sim \chi^2_{(n)}$. Then write mean and variance of χ.

12. Write the applications of χ^2 variate.

UNIT—V

13. If $t \sim t_{(n)}$, then write the mean and variance of t-distribution.

14. Write the applications of Snedecor's F-distribution.

15. Discuss how to test the significance of an observed correlation coefficient r, when population correlation coefficient $\rho = 0$.

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. State and prove Chebyshev's inequality. State Lyapunov's central limit theorem. 4+2=6

17. Write central limit theorem. State and prove De Moivre-Laplace central limit theorem. 2+4=6

UNIT—II

18. Define standard error of a statistic. Explain its utility in tests of significance for large samples. Obtain standard error of sample mean, when sample is drawn in SRSWOR. 1+2+3=6

19. Show that for a random sample of size 2 from $N(0, \sigma^2)$ population

$$E[X_{(1)}] = -\frac{\sigma}{\sqrt{\pi}}$$

Obtain the joint p.d.f. of rth and sth order statistics $X_{(r)}$ and $X_{(s)}$, $X_{(r)} < X_{(s)}$. 3+3=6

UNIT—III

20. Write the steps for testing of hypothesis. Describe the test of significance for large samples for difference between two proportions. 2+4=6

21. Define critical value and test statistic. A sample of 900 members has a mean 3·4 cm and SD 2·61 cm. Is the sample from a large population of mean 3·25 cm and SD 2·61 cm? 2+4=6

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UNIT—IV

22. Obtain moment generating function (m.g.f.) of χ^2-distribution with n degrees of freedom. For the 2×2 contingency table

a	b
c	d

prove that χ^2-test of independence gives

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}, \quad N = a+b+c+d$$

2+4=6

23. Obtain mode and skewness of χ^2-distribution with n degrees of freedom. Show that the moment generating function (m.g.f.) of $y = \log \chi^2$ is given by

$$M_y(t) = 2^t \, \Gamma\left(\frac{n}{2} + t\right) \Big/ \Gamma\left(\frac{n}{2}\right)$$ 3+3=6

UNIT—V

24. Show that for t-distribution with n degrees of freedom mean deviation about mean is given by

$$\sqrt{n}\,\Gamma\left[\frac{n-1}{2}\right] \Big/ \sqrt{\pi}\,\Gamma\left(\frac{n}{2}\right)$$

Prove that for large n, Student's t-distribution tends to standard normal distribution. Describe the procedure to test the single mean for a small sample. 2+2+2=6

25. Derive the distribution of Snedecor's F-statistic. Obtain the relation between F- and χ^2-distribution. 3+3=6

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