2023/TDC(CBCS)/EVEN/SEM/
STSHCC–202T/268

**TDC (CBCS) Even Semester Exam., 2023**

# STATISTICS

( Honours )

**( 2nd Semester )**

Course No. : STSHCC–202T

**( Algebra )**

*Full Marks* : 50

*Pass Marks* : 20

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## SECTION—A

Answer any *ten* of the following questions : 2×10=20

**1.** Define theory of equations. Write down the difference between complete and incomplete equations.

**2.** If the roots of the equation

$$x^3 + 3px^2 + 3qx + r = 0$$

are in AP, then prove that $2p^3 - 3pq + r = 0$.

3. Find the roots of the equation $x^3 - 3x^2 + 4 = 0$, if two roots of its being equal.

4. Define vector space and basis of a vector space.

5. Let $V(F)$ be a vector space. Then show that if $a, b \in F$ and $\alpha$ is a non-zero element of $V$, then $a\alpha = b\alpha \Rightarrow a = b$.

6. Prove that if two vectors are linearly dependent, then one of them is a scalar multiple of the other.

7. Define triangular matrix.

8. Define idempotent matrix and trace of a matrix.

9. Define unitary and involutory matrices.

10. Define echelon form of a matrix.

11. Define determinant of a square matrix.

12. When is a matrix said to be non-singular? Define inverse of a matrix.

13. Define rank of a matrix.





14. Prove that if $X$ is a characteristic vector of a matrix $A$, then $X$ cannot correspond to more than one characteristic value of $A$.

15. Define characteristic matrix and characteristic equation of a matrix.

## SECTION—B

Answer any *five* of the following questions : 6×5=30

16. *(a)* If $\alpha$, $\beta$, $\gamma$ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the values of *(i)* $\sum \alpha^3\beta^3$, *(ii)* $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ and *(iii)* $\sum(\alpha-\beta)^2$. 3

*(b)* State and prove the fundamental theorem of algebra. 3

17. *(a)* Solve the following by Cardan's method : 4

$$x^3 - 15x^2 - 33x + 847 = 0$$

*(b)* Find the roots of the equation

$$x^3 + 7x^2 + 14x - 9 = 0$$

if the roots are in GP. 2

18. *(a)* Show that the intersection of any two subspaces $W_1$ and $W_2$ of a vector space $V(F)$ is also a subspace of $V(F)$. 3

*(b)* Show that any subset of a linearly independent set of vectors $V(F)$ is linearly independent. 3

**19.** *(a)* State the general properties of vector space. 2

*(b)* Show that there exists a basis for each finite dimensional vector space. 4

**20.** *(a)* Show that every square matrix is uniquely expressible as a sum of symmetric matrix and a skew-symmetric matrix. 3

*(b)* Define orthogonal matrix. If

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

then show that $A$ is an orthogonal matrix. 3

**21.** *(a)* Prove that the necessary and sufficient condition for a square matrix $A$ to possess inverse is that $|A| \neq 0$. 3

*(b)* Prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^2$$

3





**22.** *(a)* Solve completely the following system of equations : 3

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$

*(b)* Prove that the system of equations $AX = B$ is consistent, i.e., it possesses a solution, if and only if the coefficient matrix $A$ and the augmented matrix $[A:B]$ are of same rank. 3

**23.** Check the consistency of the following equations :

$$x+2y-z=3$$
$$3x-y+2z=1$$
$$2x-2y+3z=2$$
$$x-y+z=-1$$

If they are consistent, solve them. 6

**24.** *(a)* Prove that the rank of the transpose of a matrix is same as that of the original matrix. 3

*(b)* Define quadratic form over a field. Write down the matrix of the following quadratic form : 1+2=3

$$x_1^2 + 2x_2^2 - 5x_3^2 - x_1x_2 + 4x_2x_3 - 3x_3x_4$$

**25.** Determine the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ 6

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