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TDC (CBCS) Even Semester Exam., 2023

STATISTICS

(Honours)

(2nd Semester)

Course No. : STSHCC–201T

(Probability and Probability Distributions)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Define discrete random variable with examples. State two properties of random variable.

2. What is distribution function of a random variable? Write down the important properties of a distribution function.

3. Explain conditional probability distribution of Y under the condition that $X = x$ in both discrete and continuous cases.

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4. Show that for a random variable X, $E(X^2) \geq \{E(X)\}^2$.

5. From the following distribution, obtain $E(X)$ and $E(X^2)$:

x	:	0	1	2	3
$p(x)$	:	1/3	1/2	0	1/6

6. Let X be a random variable with probability density function :

$$f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find $E(X)$ and $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$.

7. Define moment-generating function and characteristic function for both discrete and continuous random variables.

8. Show that characteristic function of a random variable always exist.

9. State the properties of characteristic function.

10. The mean of a binomial distribution is 40 and standard deviation is 6. Calculate n, p and q.

11. What is hypergeometric distribution?

(Continued)

12. Explain geometric distribution and obtain its mean.

13. Write the p.d.f. of beta distribution of first kind and find its mean.

14. If $X \sim U(a, b)$, then write its p.d.f. and obtain its c.d.f.

15. Define exponential distribution and find its mean.

SECTION—B

Answer any *five* questions : 6×5=30

16. The joint p.d.f. of X and Y is given by

$$f(x, y) = 4xye^{-(x^2+y^2)}; \; x \geq 0 \; ; \; y \geq 0$$

(a) Find $f(x)$, $f(y)$ i.e., marginal p.d.f. of X and marginal p.d.f. of Y. 2½

(b) Are X and Y independent? 1

(c) Find the conditional distribution of X given $Y = y$ and Y given $X = x$. 2½

17. (a) Prove that if X and Y are independent continuous random variables, then the p.d.f. of $U = X + Y$ is

$$h(u) = \int_{-\infty}^{\infty} f_X(v) f_Y(u - v) dv$$ 3

(Turn Over)

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(b) If the cumulative distribution function (c.d.f.) of a continuous random variable X is $F(x)$, then find the c.d.f. of

(i) $Y = X + a$ and (ii) $Y = X^2$.

Also find the corresponding probability density functions. 1½×2=3

18. (a) Define mathematical expectation of a random variable. State and prove the multiplication theorem of expectation. 3

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required? 3

19. (a) Show that the mathematical expectation of the sum of two random variables is equal to the sum of their individual expectations, provided all the expectations exist. 3

(b) Consider the following joint distribution of two random variables X and Y :

Y \ X	2	3	4
0	0·10	0·20	0·05
1	0·30	0·05	0·10
2	0·07	0·13	0

Find $E(X)$ and $E(Y)$. 3

20. (a) Find the effect of change of origin and scale on moment-generating function. 3

(b) Let the random variable X assume the value r with probability law,

$$P(X = r) = q^{r-1}p; \quad r = 1, 2, 3, \ldots$$

Find the m.g.f. and hence obtain mean and variance. 3

21. (a) State the uniqueness theorem of characteristic function. Prove that the characteristic function of the sum of two independent random variables is equal to the product of their characteristic functions. 1+2=3

(b) Show that except the first cumulant, all the cumulants are independent of change of origin but are not independent of change of scale. 3

22. (a) Obtain the recurrence relation for cumulants of binomial distribution with parameters n and p as

$$k_{r+1} = pq\frac{dk_r}{dp}$$ 3

(b) If X and Y are independent Poisson variates, then show that the conditional distribution of X given $X + Y$ is binomial. 3

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23. Define negative binomial distribution. Obtain the m.g.f., and hence obtain mean and variance. 6

24. *(a)* Define Weibull distribution and find its mean and variance. 1+3=4

(b) Obtain the moment-generating function of normal distribution with mean μ and standard deviation σ. 2

25. *(a)* Let X have a standard Cauchy distribution. Find the p.d.f. of X^2 and identify its distribution. 3

(b) Define Laplace distribution and obtain its characteristic function. 3

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