2019/TDC/EVEN/STSHC–201T/090

TDC (CBCS) Even Semester Exam., 2019

STATISTICS

(2nd Semester)

Course No. : STSHCC–201T

(Probability and Probability Distributions)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Define distribution function of a random variable and write down its properties.

(b) Let X and Y be two random variables with joint density function

$$f(x, y) = c(2x + y),\ 0 < x < 1,\ 0 < y < 2$$
$$= 0, \text{ otherwise}$$

Find the value of c.

(c) Explain conditional probability distribution of Y under the condition that $X = x$ in both discrete and continuous cases.

Answer *either* Question No. **2** *or* **3** :

2. *(a)* Define conditional probability density function and joint distribution function of two-dimensional random variable. 2

(b) Let X be a random variable with probability density function

$$f(x) = \frac{x}{12},\ 1 \le x \le 5$$
$$= 0, \text{ otherwise}$$

Find probability density function of $Y = 2X - 3$. 2

(c) The joint p.d.f. of (X, Y) is

$$f(x, y) = Axy,\ 1 \le x \le y,\ 1 \le y \le 2$$
$$= 0, \text{ otherwise}$$

Find the value of A. Hence obtain marginal distributions of X and Y. 1+1=2

3. *(a)* Define probability mass function and probability density function for a random variable in both discrete and continuous cases. 2

(b) Two discrete random variables X and Y have

$$P(X = 0,\ Y = 0) = \frac{2}{9}$$
$$P(X = 0,\ Y = 1) = \frac{1}{9}$$
$$P(X = 1,\ Y = 0) = \frac{1}{9}$$
$$P(X = 1,\ Y = 1) = \frac{5}{9}$$

(i) Find marginal distributions of X and Y.

(ii) Find conditional probability distribution of X given $Y = 1$.

(iii) Examine whether X and Y are independent. 2+1+1=4

UNIT—II

4. Answer any *two* of the following : 2×2=4

(a) Show that for a random variable X, $E(X^2) \ge \{E(X)\}^2$ provided that the first two moments exist.

(b) What do you mean by mathematical expectation of a random variable? Prove that $E(aX + b) = aE(X) + b$.

(c) From the following distribution, obtain $E(X)$ and $E(X^2)$:

x	:	-3	6	9
$P(x)$	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Answer *either* Question No. **5** *or* **6** :

5. *(a)* Show that the mathematical expectation of the sum of two random variables is equal to the sum of their individual expectations, provided all the expectations exist. 3

(b) The bivariate probability distribution of two random variables X and Y is given below :

$X \rightarrow$ / $Y \downarrow$	–1	0	1	*Total*
–1	0	0·1	0·1	0·2
0	0·2	0·2	0·2	0·6
1	0	0·1	0·1	0·2
Total	0·2	0·4	0·4	1

Find $E(X)$ and $E(Y)$. 3

6. *(a)* Find the mathematical expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trial. 3

(b) State and prove the multiplication theorem of expectations. 3

UNIT—III

7. Answer any *two* of the following : 2×2=4

(a) Define moment-generating function and characteristic function for both discrete and continuous random variables.

(b) If X is a random variable with probability function

$$P(X = x) = q^x p;\ x = 0, 1, 2, \cdots$$

find the moment-generating function of X.

(c) Show that characteristic function of a random variable always exists.

Answer *either* Question No. **8** *or* **9** :

8. *(a)* Find the effect of change of origin and scale on cumulants. 3

(b) State the properties of characteristic function. 3

9. *(a)* Obtain mean, variance, μ_3 and μ_4 of a random variable in terms of cumulants. 3

(b) State uniqueness theorem of moment-generating function. 1½

(c) Show that characteristic function of the sum of independent random variables is equal to the product of their respective characteristic functions. 1½

UNIT—IV

10. Answer any *two* of the following : 2×2=4

(a) The mean and variance of a binomial variate X with parameters n and p are 16 and 8. Find $P(X \geq 2)$.

(b) Give some examples of occurrence of Poisson distribution in different fields.

(c) What is hypergeometric distribution?

Answer *either* Question No. **11** *or* **12** :

11. *(a)* Obtain mean and variance of binomial distribution. 3

(b) Derive Poisson distribution as a limiting case of negative binomial distribution. 3

12. *(a)* If X follows hypergeometric distribution with parameters (N, M, n), then deduce the recurrence relation of probabilities of the distribution. 3

(b) Obtain the recurrence relation between moments of Poisson distribution

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{\partial\lambda}\right)$$

where μ_r is the rth moment about mean λ. 3

UNIT—V

13. Answer any *two* of the following : 2×2=4

(a) Write any four properties of normal distribution.

(b) Define beta distribution of first kind.

(c) Show that the exponential distribution lacks memory, i.e., if X has an exponential distribution, then for every constant $a \geq 0$, $P(Y \leq x | X \geq a) = P(X \leq x)$ for all x, where $Y = X - a$.

Answer *either* Question No. **14** *or* **15** :

14. *(a)* Show that mean and variance of Gamma distribution are equal. 3

(b) Obtain moment-generating function of normal distribution. 3

15. *(a)* If X has a uniform distribution in $(0, 1)$, find the distribution of $-2\log X$ and identify the distribution. 3

(b) If X and Y are independent Gamma variates with parameters μ and λ respectively, then show that $U = X + Y$ and $Z = \frac{X}{X+Y}$ are independently distributed. Also find distribution of U and Z. 3

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