2020/TDC (CBCS)/ODD/SEM/ STSHCC–102T/110

TDC (CBCS) Odd Semester Exam., 2020 held in March, 2021

STATISTICS

(1st Semester)

Course No. : STSHCC–102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Define limit and continuity.

(b) What is successive differentiation?

(c) What is homogeneous function?

(d) State Libnitz's theorem for successive differentiation.

(e) State maxima and minima of a function $f(x)$ at the point $x = c$.

(f) Define concavity and convexity of a function.

(g) Define Jacobian.

(h) What are stationary value and stationary point?

(i) Show that $\beta(l, m) = \beta(m, l)$.

(j) Evaluate :

$$\int_0^\infty x \cdot e^{-x} dx$$

(k) Define gamma and beta functions.

(l) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(m) What is exact differential equation?

(n) Solve :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

(o) Solve :

$$(x^2 + 2y)dx + (y^2 + 2x)dy = 0$$

(p) Write down the Clairaut's equation.

(q) Form partial differential equation by elimination of a, b from

$$z = (x - a)(y - b)$$

(r) Form partial differential equation by eliminating the function f and F

$$y = f(x - at) + F(x + at)$$

(s) Write down the Charpit's auxiliary equation.

(t) Solve :

$$p\,dq + q\,dp = 0$$

SECTION—B

Answer *any* **five** questions

2 (a) Find

$$\lim_{x \to 4} f(x)$$

when

$$f(x) = \frac{|x-4|}{x-4}, \quad x \neq 4$$
$$= 0, \quad x = 4$$

2

(b) Discuss the continuity of a function

$$f(x) = (2 + e^{-\frac{1}{x}})^{-1}$$

at $x = 0$. 3

(c) If $y = \sin^{-1} x$, then show that—

(i) $(1-x^2)y_2 - xy_1 = 0$;

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. 2+3=5

3. (a) State and prove Euler's theorem for homogeneous function. 2+2=4

(b) If

$$u = \log \frac{x^2 + y^2}{x + y}$$

then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

3

(c) If

$$f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}, \quad (x, y) \neq (0, 0)$$

$$= 0, \quad (x, y) = (0, 0)$$

find $f_x(0, 0)$, $f_{xx}(0, 0)$, $f_{xy}(0, 0)$. Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 3

4. (a) Show that the maximum value of

$$x + \frac{1}{x}$$

is less than its minimum value. 3

(b) Show that the function

$$f(x) = x^2 - 2xy + y^2 + x^2 + y^2$$

is minimum at origin. 3

(c) If

$$u = \frac{x^2 + y^2 + z^2}{x}$$

$$v = \frac{x^2 + y^2 + z^2}{y}$$

$$w = \frac{x^2 + y^2 + z^2}{z}$$

then find the value of Jacobian

$$J = \frac{\partial(x, y, z)}{\partial(u, v, w)}$$

4

5. (a) If

$$x_1 + x_2 + x_3 = u$$

$$x_2 + x_3 = uv$$

$$x_3 = uvw$$

then find

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$$

4

(b) Find the point of inflation of the curve

$$y = \frac{x^3}{a^2 + x^2}$$ 3

(c) Find for what value of x, the function

$$2x^3 - 21x^2 + 36x - 20$$

is maximum or minimum. Also find the value of the function at the maximum and minimum points. 3

6. (a) Show that

$$\sqrt{\pi} = 2^{2m-1} \frac{\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(2m)}$$ 4

(b) Prove that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$ 3

(c) Evaluate : 3

$$\int_0^\infty e^{-x^2} dx$$

7. (a) Prove that

$$\int_0^1 \frac{x^2}{\sqrt{(1-x^4)}} dx \times \int_0^1 \frac{1}{\sqrt{(1+x^4)}} dx = \frac{\pi}{4\sqrt{2}}$$ 3

(b) Prove that

$$2^n \Gamma\left(n+\frac{1}{2}\right) = 1.3.5 \cdots (2n-1)\sqrt{\pi}$$ 3

(c) Evaluate : 4

$$\iint \sqrt{4x^2 - y^2}\, dx\, dy$$

8. (a) If

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

be a function at x alone, say $f(x)$, then prove that $e^{\int f(x)dx}$ is an integrating factor of the differential equation

$$Mdx + Ndy = 0$$ 5

(b) Solve : 5

$$(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$$

9. (a) Solve : 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$$

(b) Solve : 5

$$x^2(y - py) = p^2 y$$

10. *(a)* Solve : 5

$$y^2 p - xyq = x(z - 2y)$$

(b) Find the complete integral of the equation

$$z^2(p^2 + q^2 + 1) = c^2$$

5

11. *(a)* Solve : 5

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = ax + xy / z$$

(b) Use Charpit's method to solve the equation $p^2 x + q^2 y = z$. 5

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