2021/TDC/CBCS/ODD/
STSHCC–102T/110

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

STATISTICS

(1st Semester)

Course No. : STSHCC–102T

(Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. What are function and continuity of a function?

2. Show that

$$\lim_{x \to 0} \frac{x \cdot e^{1/x}}{e^{1/x}+1} = 0$$

3. The function

$$f(x) = \frac{x^2 - 16}{x - 4}$$

is undefined at $x = 4$. What value must be assigned to $f(4)$, if $f(x)$ is to be continuous at $x = 4$?

4. Define points of inflexion of a function. Write the criteria to find such points.

5. What are stationary point and stationary value?

6. Define Jacobian.

7. Find the value of

$$\iint x^{l-1} y^{-l} e^{x+y} dx\, dy$$

extended to all positive values, subject to $x + y < h$.

8. Define gamma and beta functions.

9. Prove that

$$\int_0^\infty \frac{x^8(1 - x^6)}{(1 + x)^{24}} dx = 0$$

10. If

$$\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$$

be a function of y alone, say $\phi(y)$, then write down the integrating of $Mdx + Ndy = 0$.

11. Solve :

$$(1-x^2)\frac{dy}{dx}-2xy = x - x^3$$

12. Solve :

$$\frac{d^2y}{dx^2}-2y\frac{dy}{dx}+144y=0$$

13. Form a partial differential equation of a and b from $z = ax + a^2y^2 + b$.

14. Solve :

$$z^2 - pz + qz = 0$$

15. Solve :

$$qdp + pdq = 0$$

SECTION—B

Answer any *five* of the following questions : 10×5=50

16. *(a)* Find

$$\lim_{x \to 0} \frac{x \cdot e^x - \log(1+x)}{x^2}$$ 3

(b) What is homogeneous function of degree n? 2

(c) If

$$u = \log \frac{x^2 + y^2}{x + y}$$

then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$$ 3

(d) Define successive differentiation. 2

17. *(a)* Check the continuity of the function

$$f(x) = \begin{cases} x^2 + x, & 0 \le x < 1 \\ 2, & x = 1 \\ 2x^3 - x + 1, & 1 < x \le 2 \end{cases}$$

at $x = 1$. 3

(b) State and prove Leibnitz's theorem for successive differentiation. 1+3=4

(c) If $\log y = \tan^{-1} x$, then show that—

(i) $(1+x^2)y_2 + (2x-1)y_1 = 0$

(ii) $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$ 3

18. (a) State the necessary and sufficient conditions for a function $f(x)$ to have maximum or minimum value at a point $x = c$. 2

(b) Show that $f(x) = x^5 - 5x^4 + 5x^3 - 10$ is maximum at $x = 1$, minimum at $x = 3$ and neither at $x = 0$. 3

(c) Show that the function

$$f(x) = x^2 + 2xy + y^2 + x^3 + y^3 + x^7$$

has neither maximum nor minimum value at $x = 0$. 3

(d) Show that the function

$$f(x) = 5x^6 - 18x^5 + 15x^4 - 10$$

has three stationary points. 2

19. (a) If $u_1 = \dfrac{x_2 x_3}{x_1}$, $u_2 = \dfrac{x_1 x_3}{x_2}$ and $u_3 = \dfrac{x_1 x_2}{x_3}$, then prove that

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u_1, u_2, u_3)} = 4$$ 3

(Turn Over)

(b) If $x = r\cos\theta\cos\phi$, $y = r\sin\theta\sqrt{1-m^2\sin^2\phi}$ and $z = r\sin\phi\sqrt{1-n^2\sin^2\theta}$, $m^2+n^2=1$, then find

$$\frac{\partial(x, y, z)}{\partial(\gamma, \theta, \phi)}$$ 4

(c) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. Also find the saddle points. 2+1=3

. (a) Prove that

$$\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$ 4

(b) Evaluate : 2+2=4

(i) $\int \frac{dx}{2\sqrt{x}}$

(ii) $\int_0^1 x \cdot e^x dx$

(c) Show that $\sqrt{n+1} = n!$ 2

. (a) Evaluate : 3

$$\iint \sqrt{4x^2 - y^2}\, dx\, dx$$

(b) $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$ 4

(c) Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}}$$

22. (a) Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Solve :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

23. (a) Solve :

$$y = px + \sqrt{1+p^2}$$

(b) Solve :

$$z^2 dx + (z^2 - 2yz)\, dy + (2y^2 - yz - zx)\, dz = 0$$

24. (a) Form a partial differential equation by eliminating function f from

$$z = e^{ax+by} f(ax - by)$$

(b) Using Charpit's method, solve the equation $(p^2 + q^2)y = qz$.

25. (a) Solve : 5

$$(y^2 + z^2 - x^2)\,p - 2xyq + 2zx = 0$$

(b) Using Charpit's method, find the complete integral of the equation

$$p^2 x + q^2 y = z$$ 5

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