2022/TDC/ODD/SEM/STSHCC–102T/110

TDC (CBCS) Odd Semester Exam., 2022

STATISTICS

(Honours)

(1st Semester)

Course No. : STSHCC–102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

   (a) Define limit and continuity.

   (b) Find the value of a, if

   $$f(x) = x + a, \quad x \le 1$$
   $$= 2x \quad , \quad x > 1$$

   is continuous at $x = 1$.

   (c) What is total differentiation?

2. Answer any *one* of the following questions : 10

(a) (i) State and prove the Euler's theorem on homogeneous functions. 3

(ii) If $u = \tan^{-1} \dfrac{x^2 + y^2}{x + y}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$$ 3

(iii) If $y = \sin(m \sin^{-1} x)$, then show that

$$(1 - x^2)y_2 - xy_1 + m^2 y = 0 \text{ and}$$
$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$$ 4

(b) (i) Find the continuity of the function

$$f(x) = \begin{cases} \dfrac{1}{5 + e^{\frac{1}{x-2}}} & , \; x \neq 2 \\ \dfrac{1}{2} & , \; x = 2 \end{cases}$$

at $x = 2$. 3

(ii) Find

$$\lim_{x \to 0} \frac{\cot x - \frac{1}{x}}{x}$$ 3

(iii) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right]^2 u = -\frac{9}{(x+y+z)^2}$$ 4

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define stationary point and stationary value.

(b) State maxima and minima of a function $f(x)$ at the point $x = e$.

(c) Define convexity and concavity of a function.

4. Answer any *one* of the following questions : 10

(a) *(i)* Find the stationary points of the equation

$$f(x) = 5x^6 - 18x^5 + 15x^4 - 10$$

Find the maximum and minimum values of $f(x)$, corresponding to the obtained stationary points. 4

(ii) Find all maximum and minimum values corresponding to the function

$$f(x, y) = x^3 + y(y^2 + 12x) - 63(x + y)$$ 4

(iii) Show that the maximum value of $x + \frac{1}{x}$ is less than its minimum value. 2

(b) *(i)* Find the Jacobian of $y_1, y_2, \ldots, y_n$ being given—

$$y_1 = 1 - x_1;\ y_2 = x_1(1 - x_2),$$
$$y_3 = x_1x_2(1 - x_3),\ \ldots,$$
$$y_n = x_1x_2 \cdots x_{n-1}(1 - x_n)$$ 3

(ii) If $u = \frac{x^2 + y^2 + z^2}{x}$, $v = \frac{x^2 + y^2 + z^2}{y}$ and $w = \frac{x^2 + y^2 + z^2}{z}$.

Find the Jacobian of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ 4

(iii) Define Lagrange's multipliers for constrained optimization. 3

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(b) Find the value of $\int_0^\infty \log \frac{1}{x} \cdot e^{-x} dx$.

(c) Prove that $\beta(l, m) = \beta(m, l)$.

6. Answer any *one* of the following questions : 10

(a) (i) Prove that $\Gamma(n) = (n-1)!$. 3

(ii) Prove that

$$\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \cdot \Gamma(2m)$$ 4

(iii) Prove that

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$ 3

(b) (i) Find the value of

$$\iiint x^{-\frac{1}{2}} \cdot y^{-\frac{1}{2}} \cdot z^{-\frac{1}{2}} (1-x-y-z)^{\frac{1}{2}} dx\,dy\,dz$$

extended to all positive values of the variables, subject to the condition $x+y+z<1$. 3

(ii) Evaluate

$$\int_0^\infty e^{-x^2} dx$$ 3

(iii) Evaluate

$$\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$$ 4

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Solve $\frac{xdx - ydy}{x^2} = 0$.

(b) Solve $\frac{d^2y}{dx^2} + a^2 y = 0$.

(c) Define exact differential equation.

8. Answer any *one* of the following questions : 10

(a) (i) If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ be a fraction of x alone, say $f(x)$, then prove that $e^{\int f(x)\,dx}$ is an integrating factor of the differential equation $Mdx + Ndy = 0$. 5

(ii) Find $F(y)$, such that the total differential equation $\{(yz+z)/x\}dx - zdy + f(y)dz = 0$ is integrable. Solve it. 5

(b) Solve :

(i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}\sin x$ 5

(ii) $x^2(y - px) = p^2 y$ 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Form the partial differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$.

(b) Form the partial differential equation by the elimination of ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

(c) Write down the Charpit's auxiliary equation.

10. Answer any *one* of the following questions : 10

(a) (i) Form a partial differential equation by eliminating a, b and c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ 5

(ii) Use Charpit's method to solve the equation $p^2 x + q^2 y = z$. 5

(b) (i) Form the partial differential equation by eliminating f, g from

$$z = f(x^2 - y) + g(x^2 + y)$$ 5

(ii) Find the complete integration of the equation $z^2(p^2 + q^2 + 1) = c^2$. 5

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