2019/TDC/ODD/SEM/STSHCC-102T/115

TDC (CBCS) Odd Semester Exam., 2019

STATISTICS

(1st Semester)

Course No. : STSHCC-102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Define function and continuity of a function at a point. 1+1=2

(b) Let $f(x)=|x|$, show that $f(x)$ is not differentiable at $x=0$. 2

(c) Evaluate : 2

$$\lim_{x\to 0}\frac{e^{x}-e^{-x}-2x}{x-\sin x}$$

Answer *either* Q. No. **2** *or* Q. No. **3** :

2. *(a)* Prove that a differentiable function is always continuous. 3

(b) State and prove Leibnitz's theorem. 1+3=4

(c) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, show that

$$\frac{d^2y}{dx^2} = \frac{1}{4a}\sec^4\theta/2$$ 3

3. *(a)* If $y = \sin^3 x$, find y_n, where y_n denotes nth order differentiation of y. 3

(b) If

$$f(x) = \begin{cases} x^2 + x + 1, & 0 \le x < 1 \\ 2x + 1, & 1 \le x < 2 \end{cases}$$

check continuity and differentiability of $f(x)$ at $x = 1$. 4

(c) If $f(x) \ge 0$ and $f(x)$ is continuous at $x = c$, then show that $\sqrt{f(x)}$ is also continuous at $x = c$. 3

UNIT—II

4. Answer any *two* of the following : 2×2=4

(a) Define points of inflexion of a function. Write the criteria to find such points. 1+1=2

(b) Define concave and convex function. 1+1=2

(c) State the necessary and sufficient conditions for a function $f(x)$ to have maximum or minimum at a point $x = c$. 2

Answer *either* Q. No. **5** *or* Q. No. **6** :

5. *(a)* Find maximum and minimum value for the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. 3

(b) Show that the function

$$f(x) = x^2 + 2xy + y^2 + x^3 + y^3 + x^7$$

has neither maximum nor minimum at $x = 0$. 2

(c) If $x = r\cos\phi\sin\theta$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, prove that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$ 5

6. *(a)* Show that the curve $y = x^3$ has point of inflexion at $x = 0$. 2

(b) Find the point of inflexion, if any, of the curve

$$y = \frac{x^3}{a^2 + x^2}$$ 3

(c) Examine if $x^{\frac{1}{x}}$ possess a maximum or minimum. 3

(d) Show that maximum value of $x + \frac{1}{x}$ is less than its minimum value. 2

UNIT—III

7. Answer any *two* of the following : 2×2=4

(a) Evaluate :

$$\int \frac{dx}{2\sqrt{x}}$$

(b) Evaluate :

$$\int_0^1 xe^x dx$$

(c) What is the geometric interpretation of definite integral?

Answer *either* Q. No. **8** *or* Q. No. **9** :

8. *(a)* Show that

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cdot \cos x} = 0$$ 4

(b) Evaluate $\iint_R \sqrt{x^2 + y^2}\, dxdy$, the field of integration being R, the origin in XY-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. 6

9. *(a)* Define gamma function. Show that

$$\Gamma(x+1) = n!$$ 1+2=3

(b) State the relation between gamma and beta function. Show that $\Gamma\frac{1}{2} = \sqrt{\pi}$. 1+2=3

(c) Prove that

$$\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$ 4

UNIT—IV

10. Answer any *two* of the following : 2×2=4

(a) Define a differential equation with example. 1+1=2

(b) Define order of a differential equation. What is the order of the following differential equation? 2

$$\frac{dy}{dx} + xy = x^2$$

(c) Write the necessary and sufficient conditions for a differential equation to be exact. 2

Answer *either* Q. No. **11** *or* Q. No. **12** :

11. (a) For the given equation $y = A\cos(x+B)$, where A, B are constants, formulate differential equation. 3

(b) Solve : 3

$$x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$$

(c) Solve : 4

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

12. (a) What is integrating factor? 1

(b) Solve : 3

$$(y^2e^x + 2xy)\,dx - x^2dy = 0$$

(c) Solve : 2

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

(d) Solve : 4

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x \cdot e^{3x} + \sin 2x$$

UNIT—V

13. Answer any *two* of the following : 2×2=4

(a) Define partial differential equation. What is order of a partial differential equation?

(b) Form a partial differential equation from $x^2 + y^2 + (z-c)^2 = a^2$ by eliminating a, c.

(c) Write down the general rules for classification of partial differential equation.

Answer *either* Q. No. **14** *or* Q. No. **15** :

14. (a) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2y$, such that $z(x, 0) = x^2$ and $z(1, y) = \cos y$. 5

(b) Use Charpit's method to solve 5

$$(p^2 + q^2)y = q^2$$

15. *(a)* Solve : 5

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=au+\frac{xy}{z}$$

(b) Solve using Charpit's method

$$1+p^2=qz$$ 5

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