2023/TDC(CBCS)/EVEN/SEM/
STSDSC/GE-201T/269

TDC (CBCS) Even Semester Exam., 2023

STATISTICS

(2nd Semester)

Course No. : STSDSC/GE-201T

(Statistical Methods)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* questions : 1×15=15

1. Define discrete random variable.

2. Define cumulative distribution function of a random variable.

3. State one difference between a discrete and a continuous random variable.

4. Let the random variable X had p.m.f.

$$P(X = x) = \frac{1}{3} \; ; \quad x = -1, 0, 1$$

Find $E(X)$.

5. Define cumulant-generating function.

6. What is the effect of change of origin and scale on characteristic function?

7. State the properties of moment-generating function.

8. Fill in the blank :

   If X_1 and X_2 are independent, then $\phi_{X_1+X_2}(t) =$ _____.

9. Define marginal probability mass function.

10. What is meant by the conditional distribution of Y under the condition that $X = x$? Consider the cases where X and Y are continuous random variables.

11. Fill in the blank :

    If $p_1(x)$ and $p_2(y)$ be the marginal probability function of two independent discrete random variables X and Y, then their joint probability function $p(x, y) =$ _____.

12. Define joint probability density function.

13. What is the relation between mean and variance of binomial distribution?

14. Write the recurrence relation for the probabilities of binomial distribution.

15. Fill in the blank :

    The standard deviation of a Poisson variate is 2, the mean of the Poisson variate is _____.

16. State two properties of normal distribution.

17. State weak law of large numbers.

18. Define Chebychev's inequality.

19. State central limit theorem for independent and identically distributed variables.

20. State the assumptions necessary for the existence of CLT.

SECTION—B

Answer any *five* questions : 2×5=10

21. The function $f(x)$ is given as follows :

$$f(x) = \begin{cases} x & \text{for } 0 < x \le 1 \\ \dfrac{3-x}{4} & \text{for } 1 < x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

    Can $f(x)$ be a probability density function?

22. If X be any random variable, prove that

$$P(a \le X \le b) = F(b) - F(a)$$

23. Show that the moment-generating function of the sum of a number of independent random variable is equal to the product of their respective moment-generating function.

24. State four properties of characteristic function.

25. The p.d.f. of X and Y is given as

$$f(x, y) = 120(xy)(1 - x - y),$$
$$x, y \ge 0, x + y \le 1$$

Find the marginal probability density function of X.

26. Given

$$f(x, y) = e^{-(x+y)}, x, y > 0$$

Find conditional probability density function of Y given X.

27. Define the binomial distribution with parameters p and n.

28. Let X have a Poisson distribution with parameter $\lambda > 0$. Obtain the mean of this distribution.

29. State one similarity and one difference between weak law of large number and central limit theorem.

30. A symmetric die is thrown 720 times. Use Chebychev's inequality to find the lower bound for the probability of getting 100 to 140 sixes.

SECTION—C

Answer any *five* questions : 5×5=25

31. If X and Y are two independent random variables, then prove that

$$E(XY) = E(X) \cdot E(Y)$$

32. If X and Y are two random variables, and a, b are two constants, then prove that

$$V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab\,\text{cov}(XY)$$

33. Obtain the relation between the first four moments about mean in terms of first four cumulants.

34. Obtain the moment-generating function of Poisson distribution and hence obtain the first four cumulants of Poisson distribution.

35. Let X and Y be two random variables each taking 3 values −1, 0, 1 and the joint probability distribution as

Y \ X	−1	0	1
−1	0	0·1	0·1
0	0·2	0·2	0·2
1	0	0·1	0·1

(a) Find $E(X)$, $E(Y)$, and $V(X)$ and $V(Y)$.

(b) Are X and Y independent? Given that $Y = 0$. What is the conditional probability distribution of X?

36. Given

$$f(x, y) = \frac{2}{3}(x + 2y),\ 0 < x < 1,\ 0 < y < 1$$

Obtain the marginal distribution of X and Y and conditional probability density function of Y given X.

37. Obtain the recurrence relation for the moments of Poisson distribution. Hence find β_1 and β_2.

38. Derive the median of the normal distribution.

39. Let $p(x) = 2^{-x}$; $x = 1, 2, 3, \ldots$, prove that Chebychev's inequality gives

$$P\{|X - 2| \le 2\} > \frac{1}{2}$$

while the actual probability is $\frac{15}{16}$.

40. State and prove Bernoulli's law of large numbers.

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