2022/TDC(CBCS)/EVEN/SEM/
STSDSC/GEC–201T/125

TDC (CBCS) Even Semester Exam., 2022

STATISTICS

(2nd Semester)

Course No. : STSDSC/GEC–201T

(Statistical Method)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* questions : 1×15=15

1. Define a random variable.

2. Define sample space.

3. State the expectation of a random variable.

4. Define probability mass function of a random variable.

5. Define characteristic function of a random variable.

6. Define moment-generating function of a random variable.

7. State the limitations of moment-generating function.

8. State any two properties of characteristic function.

9. Define distribution function of a two-dimensional random variable.

10. If $F(x, y)$ is the distribution function, then what is the value of $F(-\infty, y)$?

11. Define joint density function.

12. If $f(x, y)$ is joint density function of X and Y, then find the value of

$$\int_{-\infty}^{x}\left\{\int_{-\infty}^{\infty} f(x, y)dy\right\}dx$$

13. What is the relation between mean and variance of Poisson distribution?

14. What is the moment-generating function of gamma distribution with parameter λ?

15. What is the characteristic function of $B(n, p)$?

16. Under which condition, binomial distribution tends to Poisson distribution?

17. Define Cauchy-Schwartz inequality.

18. State central limit theorem.

19. State an application of central limit theorem.

20. State strong law of large number.

SECTION—B

Answer any *five* questions : 2×5=10

21. If X and Y are two random variables and $X \leq Y$, then show that $E(X) \leq E(Y)$, provided all expectations exist.

22. Let X be a random variable with following probability distribution :

X	:	–3	6	9
$P(X = x)$	:	1/6	1/2	1/3

Find $E(X)$ and $E(X^2)$.

23. If X_1 and X_2 be two independent random variables, then show that

$$\phi_{X_1+X_2}(t) = \phi_{X_1}(t) \cdot \phi_{X_2}(t)$$

where $\phi_X(t)$ represents the characteristic function of X.

24. Show the effect of change of origin and scale on moment-generating function.

25. Define conditional distribution function and conditional probability density function.

26. The joint density function of two-dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} 2; & 0 < x < 1,\ 0 < y < x \\ 0; & \text{otherwise} \end{cases}$$

Find the marginal density of X and Y.

27. Define a Poisson random variable and find its expectation.

28. State some properties of normal distribution.

29. What is the difference between weak law of large number and strong law of large number?

30. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

SECTION—C

Answer any *five* questions : 5×5=25

31. If X and Y are two random variables for which expectation exists, then prove that

$$E(X+Y) = E(X) + E(Y)$$

32. A random variable X is distributed at random between the values 0 and 1 so that its probability density function is

$$f(x) = Kx^2(1 - x^3)$$

where K is a constant. Find the value of K. Using this value of K, find mean and variance.

33. Let X be a random variable with p.m.f.

$$p(X=x)=\begin{cases} p\cdot q^{1-x}; & x=0,1 \\ 0 \; ; & \text{otherwise} \end{cases}$$

where $p+q=1$.

Find moment-generating function of X and hence its mean and variance.

34. Prove that if X is some random variable with characteristic function $\phi_X(t)$ and if $\mu'_r = E(X^r)$ exists, then

$$\mu'_r = (i)^r \left[\frac{\partial^r}{\partial t^r}\phi_X(t)\right]_{t=0}$$

35. If X and Y are two random variables having joint density function

$$f(x,y)=\begin{cases} \frac{1}{8}(6-x-y); & 0\le x<2,\; 2\le y<4 \\ 0 \; ; & \text{otherwise} \end{cases}$$

then find—

(a) $P(X<1 \cap Y<3)$

(b) $P(X<1 \mid Y<3)$

(Continued)

36. The joint probability distribution of two random variables X and Y is given by

$$P(X=0, Y=1)=\frac{1}{3}$$

$$P(X=1, Y=-1)=\frac{1}{3}$$

$$P(X=1, Y=1)=\frac{1}{3}$$

Find the marginal distribution of X and Y.

37. If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$, then what is the distribution of $Y = X_1 + X_2$? Find its mean and variance.

38. What is memoryless property? Show that an exponential random variable lacks memory.

39. State and prove Chebychev's inequality.

40. Let $X_1, X_2, \cdots$ be iid Poisson variate with parameter λ. Use central limit theorem to estimate $P(120 \le S_n \le 160)$ where $S_n = X_1 + X_2 + \cdots + X_n$; $\lambda = 2$ and $n = 75$.

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