2023/TDC(CBCS)/ODD/SEM/ STSHCC–102T/110

TDC (CBCS) Odd Semester Exam., 2023

STATISTICS

(Honours)

(1st Semester)

Course No. : STSHCC–102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. *(a)* Define continuity of a function at a point.

   (b) What is right-hand limit?

2. Test the continuity of the function $f(x) = |x|$ at $x = 0$.

3. Define homogeneous function with an example.

UNIT—II

4. Define concave and convex functions.

5. Define Jacobian.

6. State the necessary and sufficient conditions for a function $f(x)$ to have maximum or minimum at a point $x = c$.

UNIT—III

7. Define beta and gamma functions.

8. Find the value of $\Gamma(3/2)$.

9. Show that $\Gamma(n+1) = n!$

UNIT—IV

10. Define differential equation with example.

11. Write the necessary and sufficient conditions for a differential equation to be exact.

12. Solve :

$$(x^2 - 2y)\,dx + (y^2 - 2x)\,dy = 0$$

UNIT—V

13. Define partial differential equation. What is order of a partial differential equation?

14. Form a partial differential equation by eliminating a, b from $z = (x+a)(y+b)$.

15. Solve :

$$z^2 - pz + qz = 0$$

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 10×5=50

UNIT—I

16. *(a)* Evaluate : 2

$$\underset{x \to 0}{\text{Lt}} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

(b) Give the geometrical interpretation of a derivative. 4

(c) If $u = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$, $x^2 + y^2 + z^2 \neq 0$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$ 4

17. *(a)* If $z = f(x, y)$ be a homogeneous function of degree n in x and y, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nf$$ 3

(b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$ 3

(c) State and prove Leibnitz's theorem for successive differentiation. 4

UNIT—II

18. *(a)* For what value of x, the function

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

is either maximum or minimum? Also find the maximum and minimum values of the function. 3

(b) If $x = r\cos\phi\sin\theta$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then prove that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$ 5

(c) Show that the function

$$f(x) = x^2 + 2xy + y^2 + x^3 + y^3 + x^7$$

has neither maximum nor minimum at $x = 0$. 2

19. *(a)* If

$$x_1 + x_2 + x_3 = u$$
$$x_2 + x_3 = uv$$
$$x_3 = uvw$$

then find $\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$. 4

(b) Find the point of inflation of the curve

$$y = \frac{x^3}{a^2 + x^2}$$ 3

(c) Show that the function

$$f(x) = x^2 + x\sin x + 4\cos x$$

is maximum for $x = 0$. 3

UNIT—III

20. *(a)* Prove that

$$\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$ 3

(b) Prove that

$$\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{\overline{|(m)} \overline{|(n)}}{\overline{|(m+n)}}$$

Hence show that

$$\int_0^{\pi/2} \sin^{p-1}\theta \cos^{q-1}\theta \, d\theta = \frac{\overline{|(p/2)} \overline{|(q/2)}}{2 \overline{\left|\left(\frac{p+q}{2}\right)\right.}}$$ 4

(c) Prove that

$$2^n \overline{\left|\left(n+\frac{1}{2}\right)\right.} = 1 \cdot 3 \cdot 5 \cdots, (2n-1)\sqrt{\pi}$$ 3

21. (a) Evaluate

$$\iint \sqrt{4x^2 - y^2} \, dx \, dy$$

over the integral formed by the straight line $y = 0$, $x = 1$, $y = x$. 4

(b) Prove that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$$ 6

UNIT—IV

22. (a) Solve : 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$$

(b) What is integrating factor? 1

(c) Solve : 4

$$x^2(y - py) = p^2 y$$

23. (a) Solve : 5

$$(y^2 + yz)\,dx + (z^2 + zx)\,dy + (y^2 - xy)\,dz = 0$$

(b) Solve : 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$$

UNIT—V

24. (a) Use Charpit's method to solve

$$(p^2 + q^2)y = q^2$$ 5

(b) Solve : 5

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = au + \frac{xy}{z}$$

25. *(a)* Find the complete integral of the equation

$$\pi^2 (p^2 z^2 + q^2) = 1$$ 5

(b) Solve : 5

$$(y^2 + z^2 - x^2) p - 2xyp + 2zx = 0$$

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