TDC (CBCS) Even Semester Exam., 2023

PHYSICS

(Honours)

(4th Semester)

Course No. : PHSHCC-401T

(Mathematical Physics—III)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions from the following : 2×10=20

1. Express $(1+i)$ in polar form.

2. Find the square root of i.

3. Write Cauchy-Riemann condition in polar form.

4. Define pole and order of a complex function.

5. Explain the terms 'simply' and 'multiply' connected regions.

6. Find the poles for the complex function $\cot z$.

7. Find the residue of the complex function $e^{1/z}$.

8. State and explain Cauchy integral theorem.

9. Write down the Cauchy's integral formula.

10. Define Laplace transformation.

11. Find out the Laplace transformation of 1.

12. Give the formula for Laplace transformation of derivative of a function and explain relevant terms.

13. What are the advantages of using Laplace transformation (LT) and inverse LT?

14. What is the inverse LT of $\frac{1}{(s-a)}$ when $s > a$?

15. Write down the formula for Laplace transformation of integration of a function.

SECTION—B

Answer any *five* questions from the following : 6×5=30

16. *(a)* State and prove De Moivre's theorem. 4

(b) For $z = 2 + 3i$, show $|z|$ and $\arg(z)$ in the Argand plane (i.e., Argand diagram). 2

17. Derive the necessary and sufficient conditions for a complex function to be analytic.

18. Prove Cauchy integral theorem for simply connected regions. How can this be extended for multiply connected regions? 4+2=6

19. Evaluate the following by showing all the steps :

$$\int_0^{2\pi} \frac{d\theta}{3 + 2\sin\theta}$$

20. *(a)* Find the poles and residues at the poles for the complex function $\frac{z}{\cos z}$. 3

(b) Evaluate the following integral :

$$\oint_C \frac{(1-2z)\,dz}{z(z-1)(z-2)}$$

where C is a circle given by the equation $|z| = 1 \cdot 5$. 3

21. *(a)* Prove that Laurent's expansion of a given function about a given point is unique. 3

(b) Expand the function $f(z) = \frac{1}{z(z-1)}$ in terms of Laurent's series. 3

22. State and prove two-shift theorems of Laplace transformation.

23. *(a)* What is periodic function? Give example. 1+1=2

(b) Show that Laplace transform of $e^{at} \sin \omega t$ is $\frac{\omega}{(s-a)^2 + \omega^2}$. 4

24. Write down the differential equation for the damped Harmonic oscillator. Use LT and inverse LT to solve it. 1+5=6

25. State convolution theorem. Solve the following differential equation using Laplace transformation $y'' + 9y = 9u(t-3)$; where $y(0) = y'(0) = 0$ and $u(t-3)$ is a step-unit function. 2+4=6

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