2021/TDC/CBCS/ODD/
PHSHCC–301T/150

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

PHYSICS

(3rd Semester)

Course No. : PHS HCC–301T

(Mathematical Physics—II)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Explain what you understand by odd function and even function.

2. State whether $y = \tan x$ can be expressed as a Fourier series. If so, how? If not, why?

3. State Parseval's identity.

4. Explain what you understand by regular and irregular singular points.

5. Find the ordinary point and singular point of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (1-x)y = 0$$

6. What is the degree and order of the following differential equation?

$$\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2 + y^2)xy}$$

7. Write the Rodrigues' formula for Legendre polynomial. What is the orthogonality condition of the Legendre polynomial?

8. Prove that $P_n(1) = 1$.

9. Expand $J_0(x)$.

10. Find the value of $\Gamma\left(\frac{1}{2}\right)$.

11. Prove that beta function $\beta(m, n)$ is symmetric in m and n.

12. Prove that $\delta(x) = \delta(-x)$.

13. Express Laplace's equation in cylindrical coordinates.

14. Solve

$$\frac{\partial^2 z}{\partial x \partial y} = x^2 y$$

15. Write down two applications of PDE in physics.

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$. 6

17. A sawtooth wave is defined as $f(x) = x$, $-\pi \leq x \leq \pi$. Find the Fourier series of the function. 6

18. Write down Legendre's differential equation and obtain the power series solution for it. 6

19. Discuss Frobenius method of solving a differential equation. 6

20. Find the expand of $J_{1/2}(x)$ using the general expression for Bessel function of first kind. 6

21. Prove the recurrence relations : 3+3=6

(i) $xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x)$

(ii) $2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$

22. (a) Prove that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

(b) Show that

$$\beta(m+1, n) = \frac{m}{m+n}\beta(m, n)$$ 4+2=6

23. Explain how Dirac delta function can be expressed as a limit of (a) Gaussian function and (b) rectangular function. 3+3=6

24. The displacement y of a viscously damped string is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - 2k\frac{\partial y}{\partial t}$$

Find the general solution of the above equation by the method of separation of variables. 6

25. Solve the boundary value problem

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$$

given $u(0, y) = 8e^{-3y}$, by the method of separation of variables. 6

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