2020/TDC (CBCS)/ODD/SEM/
PHSHCC–101T/147

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

PHYSICS

(1st Semester)

Course No. : PHSHCC–101T

(Mathematical Physics—I)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Find $A+B$, if

$$A=\begin{bmatrix}2&2&1\\1&5&0\\0&0&1\end{bmatrix} \text{ and } B=\begin{bmatrix}5&7&1\\0&3&0\\1&0&8\end{bmatrix}$$

(b) If A and B are symmetric matrices, then show that AB is symmetric if and only if A and B commute.

(c) Show that every square matrix can be expressed as the sum of symmetric and skew-symmetric matrices.

(d) Solve the following differential equation :

$$\frac{dy}{dx} + ay + b = 0, \; a \neq 0$$

(e) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$$

(f) Find the value of m for which the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar :

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$$
$$\vec{C} = 3\hat{i} + m\hat{j} + 5\hat{k}$$

(g) For vector $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$, find the divergence.

(h) Find the value of b for which the vector

$$\vec{A} = (2x + 3y)\hat{i} + (6y - 3z)\hat{j} + (6x - 12z)\hat{k}$$

is solenoidal.

(i) Evaluate :

$$\int_{x=0}^{1}\int_{y=0}^{2} (x^2 + 3xy^2)\, dxdy$$

(j) Find the value of

$$\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\, dxdydz$$

(k) For a given force $\vec{F} = 4xy\hat{i} - 8y\hat{j} - 2\hat{k}$, find the work done along straight line from (0, 0, 0) to (3, 1, 2).

(l) Using Gauss divergence theorem, express the Gauss law in electrostatics in differential form.

(m) Write the values of scale factors h_1, h_2 and h_3 of curvilinear coordinate system in spherical polar coordinate system.

(n) Give the expression for gradient of a scalar function ϕ in curvilinear coordinate system.

(o) Write the expression for Laplacian operator in spherical polar coordinate system.

(p) Write the expression for line and volume elements in cylindrical coordinate system.

(q) What is meant by probability? Write the expression for probability distribution function for Gaussian distribution.

(r) Define the terms 'mean' and 'variance'.

(s) Define Poisson distribution. Mention its importance in Physics. 1+1=2

(t) What are systematic and random errors? Mention various types of random errors.

SECTION—B

Answer *any* **five** questions

2. (a) What are Hermitian matrices? Show that the eigenvalues of Hermitian matrix are real. 1+3=4

(b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$ 2

3. (a) Solve the following by the method of integrating factor : 3

$$x\frac{dy}{dx} + y = x^3 + x$$

(b) Solve the following equations by matrix method : 3

$$x + 5y + 3z = 1$$
$$3x + y + 2z = 1$$
$$x + 2y + z = 0$$

4. What is gradient of a scalar function? Give its physical interpretation. Show that

$$\vec{\nabla} r^n = n r^{n-2} \vec{r}$$

where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. 1+2+3=6

5. (a) Give the physical significance of 'divergence' and 'curl'. 1½+1½=3

(b) Prove

$$\text{curl (grad } \phi) = 0$$
$$\text{div (curl } \vec{A}) = 0$$

where ϕ is a scalar and $\vec{A}$ is a vector. 1½+1½=3

6. (a) Evaluate

$$\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot \overrightarrow{dS}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant. 4

(b) Evaluate

$$\iiint_V (x^2 + y^2 + z^2)\, dxdydz$$

where V is sphere having centre at origin and radius r. 2

7. State and prove Gauss' divergence theorem. 1+5=6

8. Derive an expression for the divergence of a vector in curvilinear coordinate system. 6

9. Write the expression for the gradient of a scalar function in cylindrical coordinate system. Prove that cylindrical coordinate system is orthogonal. 2+4=6

10. (a) What is conditional probability? 2

(b) State and prove Bayes' theorem. 4

11. (a) What is hypothesis? Explain with examples, 'null hypothesis' and 'alternative hypothesis'. 1+3=4

(b) Explain the principle of least squares. 2

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