2022/TDC/ODD/SEM/PHSHCC–101T/147

TDC (CBCS) Odd Semester Exam., 2022

PHYSICS

(Honours)

(1st Semester)

Course No. : PHSHCC–101T

(Mathematical Physics—I)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—1

1. Answer any *two* of the following : 2×2=4

(a) Given

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

find x, y, z and w.

(b) If

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

find $2A + 3B$.

(c) Write the order and the degree of the differential equation

$$\frac{d^2y}{dx^2}+a^2x=0$$

2. Answer *either (a)* and *(b) or (c)* and *(d)* :

(a) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and the other anti-symmetric. 3

(b) Write the matrix A given below as the sum of a symmetric and a skew-symmetric matrix : 3

$$A=\begin{pmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{pmatrix}$$

(c) Solve the differential equation

$$(x+1)\frac{dy}{dx}=x(y^2+1)$$ 3

(d) Solve

$$x\frac{dy}{dx}+\cot y=0$$

given $y=\frac{\pi}{4}$, where $x=\sqrt{2}$. 3

UNIT—2

3. Answer any *two* of the following : 2×2=4

(a) Find a unit vector perpendicular to both of the vectors $\vec{A}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{B}=7\hat{i}-5\hat{j}+\hat{k}$.

(b) Show that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2\vec{a}\times\vec{b}$.

(c) Find the unit tangent vector at $t=2$ on the curve $x=t^2-1$, $y=4t-3$, $z=2t^2-6t$, where t is any variable.

4. Answer *either (a)* and *(b) or (c)* and *(d)* :

(a) Show that the vectors $5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are coplanar, $\vec{a}$, $\vec{b}$ and $\vec{c}$ being three non-collinear vectors. 3

(b) If $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$, find the angler which $\hat{a}$ makes with $\hat{b}$ and $\hat{c}$. 3

(c) Calculate the curl of the vector

$$xyz\hat{i}+3x^2y\hat{j}+(xz^2-y^2z)\hat{k}$$ 3

(d) Prove that for every field $\vec{v}$, div curl $\vec{v}=0$. 3

UNIT—3

5. Answer any *two* of the following : 2×2=4

(a) State Stokes' theorem.

(b) Use Green's theorem to evaluate

$$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$$

where C is the square formed by the lines $y = \pm 1$, $x = \pm 1$.

(c) State Gauss' divergence theorem.

6. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Using Green's theorem, evaluate

$$\int_C (x^2 y\,dx + x^2 dy)$$

where C is the boundary described counterclockwise of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$. 3

(b) Apply Stokes' theorem to find the value of

$$\int_C (y\,dx + z\,dy + x\,dz)$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. 3

(c) Use Gauss' divergence theorem to evaluate

$$\int_S \vec{A} \cdot d\vec{S}$$

where $A = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. 3

(d) Use Gauss' divergence theorem to evaluate

$$\iint \vec{F} \cdot d\vec{S}$$

where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, where S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 3

UNIT—4

7. Answer any *two* of the following questions : 2×2=4

(a) Write the expression for line element in cylindrical coordinate system.

(b) Write the expression for volume element in spherical polar coordinate system.

(c) Write the expression for gradient of a scalar field in spherical polar coordinate system.

8. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* :

(a) Derive an expression for divergence of the vector $\vec{F}$ in the orthogonal curvilinear coordinate system. 3

(b) Find gradient of $\phi = xyz$ in cylindrical coordinate system. 3

(c) Find the location of the positive roots of $x^3 - 9x + 1 = 0$, and evaluate the smallest one by bisection method correct to two decimal places. 3

(d) Evaluate $f(1 \cdot 2)$ from the following data using Newton's forward interpolation formula : 3

x	0	1	2	3	4
y	1	1·5	2·2	3·1	4·3

UNIT—5

9. Answer any *two* of the following questions : 2×2=4

(a) What do you mean by independent random variables?

(b) Explain random errors with examples.

(c) Define the terms *(i)* mean and *(ii)* variance.

10. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* :

(a) Explain the basic rules for propagation of errors. 3

(b) If the physical quantities, a, b, y and z are related by the expression

$$x = \frac{3a^k b^p}{y^k z^r}$$

where k, p, r are indices. Find the expression of relative error of x. 3

(c) What is binomial distribution? Write two properties of binomial distribution. 1+2=3

(d) Explain conditional probability. A disc is thrown twice and the sum of numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once? 1½+1½=3

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