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2021/TDC/CBCS/ODD/
PHSHCC–101T/147

**TDC (CBCS) Odd Semester Exam., 2021**
**held in March, 2022**

# PHYSICS

**( 1st Semester )**

Course No. : PHSHCC–101T

**( Mathematical Physics—I )**

*Full Marks* : 50
*Pass Marks* : 20

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## SECTION—A

Answer any *ten* of the following questions : 2×10=20

**1.** Find the values of $x$, $y$ and $z$ which satisfy the matrix equation

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$$

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**2.** If

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$$

then obtain the product $AB$.

**3.** What do you mean by 'order' and 'degree' of a differential equation?

**4.** Find the area of the parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$.

**5.** Find the volume of the parallelopiped if $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -3\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the three coterminous edges of the parallelopiped.

**6.** At any point of the curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$, find the tangent vector.

**7.** If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the $xy$-plane from (0, 0) to (1, 4) along a curve $y = 4x^2$, then find the work done.

**8.** Evaluate by Stokes' theorem

$$\oint_C (yzdx + zxdy + xydz)$$

where $C$ is the curve $x^2 + y^2 = 1$, $z = y^2$.



( Continued )



**9.** State Green's theorem.

**10.** Write the expression for line element in spherical polar coordinate system.

**11.** Write the expression for volume element in cylindrical coordinate system.

**12.** Write the expression for gradient of a scalar function in cylindrical coordinate system.

**13.** Define the term 'constant error' and give a suitable example.

**14.** The temperatures of two bodies measured by a thermometer are given by $T_1 = (20 \pm 0 \cdot 5)°C$ and $T_2 = (50 \pm 0 \cdot 5)°C$. Calculate the temperature difference and error therein.

**15.** What do you mean by least square fit method?

## SECTION—B

Answer any *five* of the following questions : 6×5=30

**16.** *(a)* If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

then show that $A^2 - 4A - 5I = 0$ where $I$ and 0 are unit matrix and null matrix of order 3 respectively. 3



( Turn Over )

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(b) If

$$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

then prove that $A^{-1} = A'$, $A'$ being the transpose of $A$. 3

**17.** (a) Solve the following differential equation by the method of integrating factor

$$(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$$ 3

(b) Solve the differential equation

$$(2xy + x^2)dy = (3y^2 + 2xy)dx$$ 3

**18.** (a) Find $m$ so that the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} - m\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar. 3

(b) Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Find the vector $\vec{a} \times (\vec{b} \times \vec{c})$. 3

**19.** (a) If $\frac{d\vec{a}}{dt} = \vec{u} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{u} \times \vec{b}$, then prove that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{u} \times (\vec{a} \times \vec{b})$. 3

(b) If $\phi = 3x^2y - y^3z^2$, then find grad $\phi$ at the point $(1, -2, -1)$. 3





**20.** (a) If $\vec{F} = 2y\hat{i} - z\hat{j} + x\hat{k}$, then evaluate $\int_C \vec{F} \times d\vec{r}$ along the curve $x = \cos t$, $y = \sin t$, $z = 2\cos t$ from $t = 0$ to $t = \frac{\pi}{2}$. 3

(b) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2$, $z = 0$. 3

**21.** (a) Using Green's theorem, evaluate $\int_C (x^2 y dx + x^2 dy)$ where $C$ is the boundary described counter-clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1). 3

(b) Using Stokes' theorem, evaluate $\int_C [(2x - y)dx - yz^2 dy - y^2 z dz]$ where $C$ is the circle $x^2 + y^2 = 1$ corresponding to the surface of the sphere of unit radius. 3

**22.** (a) Find the expression for $\nabla^2\phi$ in orthogonal curvilinear coordinate system. 3

(b) If $u = 2x + 3$, $v = y - 4$, $w = z + 2$ and $\vec{r}$ be the position vector, i.e., $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $\frac{\partial \vec{r}}{\partial u}$, $\frac{\partial \vec{r}}{\partial v}$ and $\frac{\partial \vec{r}}{\partial w}$ are mutually orthogonal. 3

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23. *(a)* Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$. 3

*(b)* Evaluate

$$I = \int_0^1 \frac{1}{1+x} dx$$

correct to three decimal places using Simpson rule with $h = 0 \cdot 125$. 3

24. *(a)* Discuss the different types of systematic errors associated with a measurement. 3

*(b)* What is random error?

The period of oscillation of a simple pendulum is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

The measured value of $L$ is 20·0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 sec using a wrist watch of 1 sec resolution. What is the accuracy in the determination of $g$? 1+2=3

25. *(a)* Define standard error and probable error. What are the rules of testing the significance of correlation? 2+1=3





*(b)* If two resistors of resistances

$R_1 = (5 \pm 0 \cdot 1)$ ohm and $R_2 = (10 \pm 0 \cdot 2)$ ohm

are connected in *(i)* series and *(ii)* parallel, then find the equivalent resistance in each case with limits of percentage error. 1½+1½=3

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