2018/TDC/ODD/PHYC–101T/054

TDC (CBCS) Odd Semester Exam., 2018

PHYSICS

(1st Semester)

Course No. : PHSHCC–101T

(Mathematical Physics—I)

$$\frac{\text{Full Marks : 50}}{\text{Pass Marks : 20}}$$

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

UNIT—I

1. Explain any *two* of the following with examples : 2×2=4

   (a) Hermitian matrix

   (b) Orthogonal matrix

   (c) Unitary matrix

2. Answer (a) or (b) :

   (a) Find the eigenvalues and eigenvectors of the following matrix : 6

   $$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(b) **(i)** Find the integrating factor and hence solve the differential equation

$$\frac{dy}{dx} + xy = 2x \qquad 3$$

(ii) Solve the differential equation

$$9\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + y = 0$$

Also find its Wronskian to show that its two solutions are independent. 3

UNIT—II

3. Answer any *two* questions : 2×2=4

(a) If
$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$
show that $\vec{A}$ and $\vec{B}$ are mutually perpendicular.

(b) Find the unit vector perpendicular to each of the vectors $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$.

(c) A particle moves from point $(4, -3, -5)$ metre to point $(-1, 4, 3)$ metre under the action of force $\vec{F} = (-3\hat{i} - \hat{j} + 2\hat{k})$ N. Find the work done by the force.

4. Answer *(a)* and *(b)* or *(c)* and *(d)* :

(a) Show that the scalar product of two vectors is invariant under the rotation of coordinate axes. 3

(b) Prove that
$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0 \qquad 3$$

(c) What is the gradient of a scalar point function? Give its geometrical interpretation. 2+2=4

(d) Show that
$$\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$$
is a solenoidal vector. 2

UNIT—III

5. Answer any *two* questions : 2×2=4

(a) Use polar coordinates to evaluate the surface integral $\iint (x^2 + y^2)\,dx\,dy$ over the first octant of the circle $x^2 + y^2 = a^2$.

(b) Find the total mass of the body in the region $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z \le 2$ with density function $\sigma(x, y, z) = xyz$.

(c) Find the directional derivative of $\phi = x^3 + y^3 + z^3$ at the point $(1, -1, 2)$ in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.

6. Answer *(a)* or *(b)* :

(a) State and prove Gauss' divergence theorem!   6

(b) Verify Stokes' theorem for the vector $\vec{A} = (3x - 2y)\hat{i} + x^2 z\hat{j} + y^2 (z+1)\hat{k}$ for a plane rectangular area having vertices at (0, 0), (1, 0), (1, 2), (0, 2) in the *xy*-plane.   6

UNIT—IV

7. Answer any *two* questions :   2×2=4

(a) What is orthogonal curvilinear coordinate system?

(b) Write the expression for gradient of a scalar in orthogonal curvilinear coordinates.

(c) Write the expression for line element and volume element in orthogonal curvilinear coordinates.

8. Answer *(a)* or *(b)* :

(a) Derive the expression for divergence of a vector in terms of orthogonal curvilinear coordinates.   6

(b) What is cylindrical coordinate? Derive the expression for gradient of a scalar in cylindrical coordinate system.   6

UNIT—V

9. Answer any *two* questions :   2×2=4

(a) Marks obtained by the students of a class of 30 numbers out of total 100 marks are as given below :

Marks obtained	No. of students
60	4
70	6
80	3
85	7
90	8
95	2

Find the mean, median and mode of the above given data.

(b) Find the standard deviation of the following set of data :

4, 6, 8, 4, 10

(c) Write the probability distribution function for a binomial variate. Under which condition, binomial distribution tends to Poisson's distribution?

10. Answer *(a)* or *(b)* :

    (a) Find the expression for mean and standard deviation of binomial distribution. 6

    (b) Show that mean and variance are equal in Poisson's distribution. 6

★★★