TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(6th Semester)

Course No. : MTMSEC–601T

(Analytical Geometry)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* of the following as directed : 1×15=15

1. Find the equation of the straight line $\frac{x}{a}+\frac{y}{b}=2$, when the origin is transferred to the point (a, b).

2. Write down the equation of external bisectors of the angles between the pair of straight lines of $ax^2+2hxy+by^2=0$.

3. Prove that the equation
$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$
represents a pair of perpendicular straight lines.

4. Every homogeneous equation of second degree represents ——. (Fill in the blank)

5. Define radical axis.

6. What is rectangular hyperbola?

7. Write down the parametric equation of a circle.

8. Write down the position of a point (1, 2) with respect to the circle $x^2 + y^2 = 25$.

9. What do you mean by eccentricity of a conic?

10. When is a straight line said to be a normal to a conic?

11. What is the condition of tangency of a straight line $y = mx + c$ to a parabola $y^2 = 4ax$?

12. Write down the equation of a normal to the conic $\frac{l}{r} = 1 + e\cos\theta$.

13. When is a circle said to be a great circle?

14. Define skew line.

15. What do you mean by shortest distance between two lines?

16. Write down the equation of a sphere when (x_1, y_1, z_1) and (x_2, y_2, z_2) are the extremities of its diameter.

17. What do you mean by guiding curve of a cylinder?

18. Define cone.

19. Write down the general equation of second degree in x, y, z.

20. Define the term right circular cylinder.

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove that $pq = -1$.

22. Transform to axes inclined at 30° to the original axes the equation

$$x^2 + 2\sqrt{3}xy - y^2 = 8$$

23. Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point (2, –4, 6).

24. Find the condition that the straight lines $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, $\frac{x}{a\alpha} = \frac{y}{a\beta} = \frac{z}{a\gamma}$ and $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ will lie on a plane.

25. Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.

26. If the normal at the point $(at_1^2, 2at_1)$ on the parabola $y^2 = 4ax$ meets it again at the point $(at^2, 2at)$, show that $t = -t_1 - \frac{2}{t_1}$.

27. Write down the equation of the plane containing the first line and the line of shortest distance between the lines

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \text{ and}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

28. Find the equation of the circle which passes through the points (3, 4) and (3, –6) and which has its centre on the straight line $2x + 3y = 3$.

29. Find the equation of a right circular cylinder of radius 3 and whose axis is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$$

30. Write down the condition for general equation of second degree to represent cone.

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Show that the area of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is

$$\frac{\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$$

Find the length of the side which lies on the straight line $lx+my=1$. 3+2=5

32. The expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ by rotation of rectangular axes about origin, then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$.

33. *(a)* Find the radical axis of the two circles $x^2+y^2+4x-2y+9=0$ and $x^2+y^2+2x+3y-5=0$. 2

(b) Prove that the circle $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ will touch each other if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$. 3

34. *(a)* Prove that the center of the three circles $x^2+y^2=1$, $x^2+y^2+6x-2y=1$ and $x^2+y^2-12x+4y=1$ lie on a straight line. 3

(b) Find the equation of the circles which cuts orthogonally each of the three circles $x^2+y^2+2x+17y+4=0$, $x^2+y^2+7x+6y+11=0$ and $x^2+y^2-x+22y+3=0$. 2

35. Find the polar equation of a conic referred to a focus as pole. When will this conic be a parabola? 4+1=5

36. *(a)* Show that the straight line $x\cos\alpha+y\sin\alpha=p$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$. 3

(b) Show that three normals can be drawn from a given point to a parabola. 2

37. Prove that the circles

$$2(x^2+y^2+z^2)+8x-13y+17z-17=0,$$
$$2x+y-3z+1=0 \text{ and}$$
$$x^2+y^2+z^2+3x-4y+3z=0,\ x-y+2z-4=0$$

lie on the same sphere. Find its equation. 3+2=5

38. Find the shortest distance between the straight lines

$$\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1} \text{ and } \frac{x+3}{3}=\frac{y+7}{-2}=\frac{z-6}{-4}$$

and the equation of the line of shortest distance. 2+3=5

39. *(a)* Find the equation of the cylinder generated by straight lines parallel to $\frac{x}{1}=\frac{y}{5}=\frac{z}{-2}$ the guiding curve being the conic $x=0,\ y^2=6z$. 3

(b) Find the equation of the cone whose vertex is the origin and base is the circles $x=a,\ y^2+z^2=b^2$. 2

40. *(a)* A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$\left(\frac{b}{c}+\frac{c}{b}\right)yz+\left(\frac{c}{a}+\frac{a}{c}\right)zx+\left(\frac{a}{b}+\frac{b}{a}\right)xy=0$$ 3

(b) Find the equation of the right circular cylinder of radius 3 and whose axis is $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z-3}{6}$. 2

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